Barnard; Vice-chairman, W. C. Krathwohl; Secretary-Treasurer, C. N. Mills; and Associate Secretary-Treasurer, Edith I. Atkin. The meeting for the year 1932 will be at the University of Illinois.

The following seven papers were presented:

1. "Reorganization of material for freshman mathematics" by Professor Mayme I. Logsdon, University of Chicago.

2. "Real branches of algebraic curves" by Professor H. R. Brahana, University of Illinois, by invitation.

3. "On some points relating to the restricted theory of relativity" by Dr. Luise Lange, Crane Junior College.

4. "Stellar systems" (Evening lecture) by Professor W. D. MacMillan, University of Chicago.

5. "Analysis Situs" by Dr. N. E. Rutt, Northwestern University, by invitation.

6. "An experimental study of the effects of sectioning in college mathematics" by Miss Helen Taylor, University of Illinois, by invitation.

7. "Some recent work on the application of Padé approximants to divergent series" by Dr. H. S. Wall, Northwestern University, by invitation.

Abstracts of six of these papers follow:

1. A reorganization of the material of freshman mathematics is to be desired because of certain weaknesses in the present almost universal practice of allowing three (or four) semester hours or five quarter hours to each of the pre-calculus subjects. These weaknesses are, (1) the inequitable distribution of time to the three subjects, trigonometry, college algebra, and analytical geometry; (2) the ineffectiveness of teaching an unrelated succession of topics in algebra; (3) the loss in time and in understanding by presenting without the notions of the calculus topics which should properly be developed from that point of view; (4) the unavoidable duplication due to the fact that certain topics and certain methods belong to more than one of the subjects.

The arrangement described by Professor Logsdon is believed by her to be free from the above objections, to present the material in a logical manner, and to secure and hold the interest of the students because they can see from the beginning that they are building a structure in which each part has a definite relation to the whole.

2. Mr. Brahana gives an exposition of the representation of an algebraic curve by means of a closed surface in a Euclidean three-space and discusses the location of the real part of the curve on this surface. Some curves of orders three and four are considered in detail.

3. The purpose of the paper is to point out how, in an introductory presentation of the theory of relativity, greater emphasis on certain basic ideas and greater consideration for the reader's preconceived notions may be helpful in achieving better understanding and preventing misconceptions. Two points are chosen for discussion: (a) on time and time determination; (b) on the unique role played by the velocity of light in this theory.

(a). The distinction is drawn between psychological and physical time; principles of measurement for either are discussed; these introduce subclassification of physical time as that of "nearby" and "distant" events. The relevancy of these distinctions is maintained against arguments to the contrary. A new classification is then obtained: "local" time, which is psychological time plus the time of nearby events, and "distant" time. It is further pointed out that, in applying the Lorentz transformation, judgment has to be used because in the formula the above distinctions are not apparent; and that some erroneous results have been derived in such manner, e.g. that an observer *sees* the lengths in a moving system contracted, and the derolement of time retarded. Correctly applied the Lorentz transformation yields the result that an observer sees an approaching object stretched out, and a receding one contracted; an approaching clock run faster, and a receding one run slower, in the ratio $[(c+v)/(c-v)]^{1/2}$: 1. It is further shown that these results followed already from the classical theory, only with the respective ratios $(1-v^2c^{-2})^{-1/2}$ times greater; and that in the Jupiter system such a clock, alternatingly approaching and receding from the earth, is available to observation. The difference between the effects calculated from the one and other theory is too small to afford an experimental test, being of the order $8 \cdot 10^{-4}$ seconds only.

(b). (1) the idea of a "transcendental simultaneity" is discussed and evaluated; (2) the idea of establishing distant simultaneities by carrying synchronized clocks from place to place is shown to be intrinsically inapplicable. (3) the unique role played by light as a signal is shown to spring from the concurrence of two facts of immediate or indirect experience: that it is the greatest signal velocity known, and that it is said to be constant relative to systems in uniform relative motion. In conclusion it is pointed out that the postulate of the constancy of the velocity of light can and should likewise be treated with sufficient care so as to prevent the impression of its being contrary to sound reason.

5. This is an expository paper describing briefly the theory of dimensionality the separation of spaces by means of sums of their connected subsets, and the notion of a curve.

6. The mathematics department at the University of Illinois conducted a four-year experiment in sectioning students in analytic geometry on the basis of their grades in college algebra and trigonometry. Sixty-six sections including sixteen hundred students and fifty-five different instructors were involved in the experiment.

When three sections met at one hour students were divided into high, middle, and low sections. When two sections met at an hour students were put into high and low sections. Control sections were maintained in which no sectioning was attempted. Students from the experimental and control sections who made equal scores on the Iowa placement tests were considered to be of approximately equal ability and were then compared at the end of the semester on the basis of a new type test in analytics and an old type test in analytics. Through the four years, the students sectioned into three ability groups averaged 7% higher scores

on the old type test and 5% higher on the new type test on the basis of a perfect score. In terms of the average score for the 1600 students the gain is $12\frac{1}{2}\%$. Students sectioned into two groups at an hour show a 4% gain on the old type test, but less than 1% on the new type test. The experiment affords strong evidence for the value of sectioning when three groups meet at an hour and some evidence for sectioning two at an hour. The method of sectioning has been adopted as a policy in the department at the University of Illinois for classes in analytics.

7. Dr. Wall shows that the summability of a power series $\Sigma c_i z^i = P(z)$ afforded by the Padé table is a species of the following kind of summability. Let

$$y_s = \sum_{i=0}^{s} c_i z^i + z^{s+1} \sum_{i=0}^{\infty} c_i^{(s)} z^i,$$

where

$$\sum_{i=0}^{s} \alpha_{i,p}^{(s)} c_i + \sum_{i=0}^{p} \alpha_{s+i+1,p}^{(s)} c_i^{(s)} = 0, \; p = 0, 1, 2, \cdots .$$

Now, if, over a region R of the z-plane, $\operatorname{Lim}_s y_s = y$, then y is a generalized sum (over R) of $P(z)$. For a proper choice of the $\alpha_i^{s}{}_{,p}$, y_s is a Padé approximant of $P(z)$, and is a rational fraction. This "Padé summability" is discussed from various angles, e.g. regularity, applicability, connection with other summabilities, etc. The writer's notion (Bulletin of the American Mathematical Society, vol. 36, p. 646) of a straight line of functional equivalents of $P(z)$ is discussed and amplified.

C. N. MILLS, *Secretary*

THE EIGHTH ANNUAL MEETING OF THE INDIANA SECTION

The eighth annual meeting of the Indiana Section of the Mathematical Association of America was held at Ball State Teachers College, Muncie, Indiana, on Friday and Saturday, May 1 and 2, 1931.

There were one hundred fifty present at the meeting, including the following twenty-five members of the Association: W. C. Arnold, E. R. Bowersox, G. E. Carscallen, P. T. Copp, H. T. Davis, W. E. Edington, P. D. Edwards, T. C. Fry, E. D. Grant, H. E. H. Greenleaf, F. H. Hodge, H. K. Hughes, Florence Long, Juna M. Lutz, H. R. Mathias, T. E. Mason, H. A. Meyer, T. W. Moore, Mary S. Paxton, J. A. Reising, C. K. Robbins, L. S. Shively, W. O. Shriner, R. O. Virts, K. P. Williams.

On Friday evening at 6:30 a banquet was held at Lucina Hall on the campus which was attended by 58 members and guests of the Association. Professor L. H. Whitcraft of the mathematics department of Ball State Teachers College presided. Following two musical selections, the keys of the College were turned over to the visitors in a felicitous address by President L. A. Pittenger. The welcome

was further emphasized in a short address by Dean Ralph Noyer who spoke of the great debt owed to mathematicians by the world as exemplified in the work of Clerk Maxwell and others who have connected pure mathematics with the world of our experience.

At eight o'clock a public lecture was given in Science Hall by Dr. Thornton C. Fry, of the Bell Telephone Laboratories of New York City, who spoke on the subject, "Mathematics comes into its own." Dr. Fry began by painting a picture of the history of mathematical physics from its origin in the atomic theory of Democritus. He showed in epitome the long warfare between proponents of the corpuscular theory of nature and advocates of the theory of continuity in the underlying stratum of things. Bringing the story to the modern era he showed the great perplexity of physics today in its attempt to rationalize the experiments of the quantum theory. The speaker affirmed that we have had three major syntheses of physical experience: (1) Newton's theory of universal gravitation; (2) Maxwell's theory of electricity and magnetism; (3) Einstein's geometrization of nature. He predicted that the fourth synthesis will appear in the mysteries of the quantum theory and stated that great progress toward this end appears in the work of Schrödinger, Born, de Broglie, Heisenberg, Dirac and other mathematical physicists. He emphasized the fact that the first three syntheses were of a definitely mathematical character and that there were indications that the fourth would be of the same nature. The speaker asserted that mathematics only fully comes into its own in so far as it comes in intimate contact with more objective sciences such as physics, quoting in this connection from the preface to the second edition of Newton's *Principia:* "Those who fetch from hypotheses the foundations on which they build their speculations, may form, indeed, an ingenious romance; but a romance it will still be." He concluded with a plea for more attention in America to this border-line work which has heretofore been left almost exclusively to Europeans.

The session on Saturday morning in Science Hall was presided over by Professor P. D. Edwards, Ball Teachers' College, chairman. The following officers were elected: Chairman, Professor G. E. Carscallen, Wabash College; Vice-chairman, Professor T. E. Mason, Purdue University; Secretary-Treasurer, Professor H. T. Davis, Indiana University.

A chairman's address was made by Professor Edwards on "Reorganization of secondary mathematics." Professor Edwards discussed the advantages that would result from a reorganization of secondary mathematics on the plan recommended by the National Committee. It was pointed out that at present 405 of the 841 high schools in Indiana are organized on the 6–6 plan. Mathematics is a required subject through grade nine, but is an elective in grades 10–12. Comparison was made with the mathematics curriculum in Europe where algebra and intuitive geometry are begun as early as grade six. The advantages of the longer period of instruction in algebra were emphasized. The reorganization results in advantages to the student who does not continue his mathematical studies beyond grade nine as well as to the student who continues his study in the senior

high school and college. Emphasis was placed on the rapid increase in recent years of the need for knowledge of statistics, graphs, equations, compound interest and annuities. For the person continuing in mathematics it was pointed out that the three-year interval for the acquisition of the essentials of algebra would result in better fixation of habits of thinking in terms of algebraic symbols. Suggestions were made concerning possible reorganization of material in the senior high school. It was shown that in the decade since the National Committee made its report there has been practically no change in the teaching of secondary mathematics in Indiana. A plea was made that the members of the Mathematical Association take the initiative in bringing about such a reorganization as will best serve the interests of the state.

Mr. Russell Sullivan of Indianapolis presented by invitation an illustrated lecture on "The evolution of the stars." Mr. Sullivan devoted special attention to modern interpretations of the various kinds of nebulas. He sketched the step-ladder evolutionary theory which puts the red giant stars at one foot of the ladder (youth), the blue stars at the top, and the red dwarfs at the other foot (old age). He concluded with an exposition of the interpretation of the high recessive velocities of the spiral nebulas as evidence of the curvature of space-time.

The remainder of the program consisted of the following papers, the third one being read by title:

1. "The early history of Kepler's equation" by Professor K. P. Williams, Indiana University.

2. "Class size, past, present and future" by Mr. C. E. Trueblood, Arsenal Technical Schools, Indianapolis, by invitation.

3. "Synthetic projective geometry as an aid to high school teachers" by Professor W. C. Arnold, De Pauw University.

4. "Some applications of the calculus of residues to the theory of functions" by Professor H. K. Hughes, Purdue University.

5. "Notes on generatrix functions with an application" by Mr. Fred Robertson, Iowa State College, Ames, Iowa, by invitation.

6. "A problem in grade distribution" by Professor C. K. Robbins, Purdue University.

7. "Some recent results in the theory of elimination" by Professor T. W. Moore, Indiana University.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. This paper described Kepler's formulation of his famous problem and his treatment of it as given in the *Astronomia Nova* (1609), the *Epitome Astronomiae Copernicae* (1618), and the *Tabulae Rudolphinae* (1627). The tabular solution in the latter disposed of all the actual astronomical necessities of the time, but the comments in the *Astronomia Nova*, which despaired of the possibility of a solution, and the method of approximations in the *Epitome*, usually overlooked by commentators, showed Kepler's desire for an adequate treatment. The later his-

tory of the problem was traced to the time of Lagrange. It was pointed out how the problem could be made an instructive one in a course on the history of mathematics that drew on actual sources.

2. In 1924–25 the speaker established large classes with the idea of developing a suitable technique for handling large numbers in mathematics, in view of the conclusion reached from educational studies made since 1896 that size of class has little influence upon student achievement. His classes ranged in size from 80 to 120. The technique which he established in his first four or five classes has been used with minor improvements ever since. He is now teaching his seventeenth class of 100 in mathematics. His conclusions are: (1) In spite of the success of large class technique there will always be a place for the small class; (2) one teacher will be able to teach three classes of 100 students as efficiently as six classes of 25; (3) one serious fault of present teaching technique is that highly trained teachers are required to perform too many minor details. This can be eliminated by improved methods.

3. In this paper a plea is made for the wider study of synthetic projective geometry among high school teachers. It is pointed out that this mathematical discipline is self-contained and hence is admirably adapted to the needs of one who might wish to extend his knowledge without class-room instruction. Synthetic projective geometry is also more closely connected with the geometry in which the high school teacher is giving instruction than with either analytics or calculus, and its range of beautiful theorems has an esthetic appeal that is not exceeded by other mathematical subjects of similar difficulty.

4. In this paper Professor Hughes considered functions defined by certain types of infinite series, the series themselves being regarded as given. The particular problem was to extend analytically the function defined by the given series into regions exterior to the region of convergence of the series. As consequences of the results obtained, certain further results pertaining to the asymptotic development of the functions in question were discussed. Reference was made to results already established by Barnes, Ford, and others regarding functions defined by power series. The speaker considered functions defined by factorial series of the first and second kinds, and by Dirichlet series. The methods of the calculus of residues was employed to solve the problem of analytic extension. Some asymptotic properties were also obtained.

5. If z^{-n}, where $z=d/dx$, is an n-fold integral operator, it can be shown that this operator is equivalent to another, i.e., $x^nQ_n(\mu)$, $\mu=xz$, which has a Taylor's expansion about $\mu=0$. This operator is called a *generatrix function*. Both μ^{-n} and $Q_n(\mu)$ may be shown to satisfy the differential equation: $\mu Q''(\mu)+(n+\mu+1)Q'(\mu)+nQ(\mu)=0$. The object of the present paper is to show that the solution of the equation of heat conduction, expressed in spherical coordinates, r, θ, ϕ, is given by a function of the form: $V=F(r, t)\Theta(\theta)\Phi(\phi)Q_n(\nu)$, where $\nu=4r^2kt$.

6. Professor Robbins presented a solution of the following problem: The grades used in a certain institution are A, B, C, and D. Suppose that the distri-

bution of grades for a certain period of time was A', B', C', D', where A' is the number of A grades etc. The grades are redefined in such a manner that it is estimated that the distribution would have been A'', B'', C'', D'', if the new definitions had been in effect during the above period of time. Suppose that the distribution of an individual for this same period was a', b', c', d'. What would the distribution of this individual necessarily have been under the new definition of grades?

7. This paper was a brief resumé of the results contained in two papers published recently in the Annals of Mathematics under the titles: "Extended results in elimination," (vol. 30, pp. 92–100), and "On the resultant of two binary forms," (vol. 31, pp. 185–189). They are concerned with the problem of representing the eliminant of a definite number of the forms, where the number depends upon the dimension of the domain of definition, as a single determinant free of extraneous factors. In the first paper the question of forms in more than one set of variables was considered, and in the second, new forms of the resultant determinant of two binary forms were exhibited.

At the afternoon session of the Section a resolution was adopted expressing the sorrow of the members at the news of the sudden death of Professor W. A. Zehring of Purdue University, "who has been a faithful attendant of the Section meetings, and a zealous and enthusiastic teacher of mathematics for the past quarter of a century." A second resolution was adopted expressing the appreciation of the members of the Section for the hospitality and courtesy extended to them by President Pittenger and the members of the mathematics department of Ball State Teachers College.

In close connection with the meetings of the Section a conference on the teaching of high school mathematics was held. The conference joined with the Section for the morning program, but met separately in the afternoon.

H. T. DAVIS, *Secretary*

THE ANNUAL MEETING OF THE NEBRASKA SECTION

The annual meeting of the Nebraska Section of the Association was held at Lincoln on May 8, 1931, jointly with the mathematics section of the Nebraska Academy of Sciences. Thirty persons were in attendance, including the following members of the Association: M. A. Basoco, A. K. Bettinger, W. C. Brenke, C. C. Camp, A. L. Candy, M. M. Flood, M. G. Gaba, A. L. Hill, J. M. Howie, R. M. McDill, T. A. Pierce, Lulu L. Runge. Mr. M. M. Flood, chairman of the Section, presided.

The following officers were elected for the ensuing year: Chairman, Prof. A. K. Bettinger, Creighton University, Omaha; Secretary, Prof. A. L. Hill, Peru State Teachers College; Treasurer, Prof. J. M. Howie, Nebraska Wesleyan University.

The program at the joint meeting was as follows:

1. "On generalized differentiation" by R. M. Ely.
2. "Study of some new orthogonal polynomials" by Mrs. Madeline Grenard.
3. "Sylvester's theorem and an application to the Tschirnhaus transformations" by Violet Wochner.
4. "Demonstration of the Mader harmonic analyser" by A. P. Cowgill.

1. Following a definition given by H. T. Davis in the American Journal, 1924, and with the restriction that $\phi(c)=0$, Mr. Ely studied the operation ${}_cD_x^{v+m}(x)$, $0<v<1$, $m=1, 2, 3, \cdots$; extended the "rationalizing" process of Davis to differential equations of fractional order higher than one, and worked out some examples.

2. Mrs. Grenard discussed polynomials orthogonal to x^{2r}, in the interval $(-1, +1)$ and also when this interval is replaced by a simple closed curve in the complex domain. Difference equations similar to that for the nth Legrendre polynomial were obtained. Corresponding differential equations do not exist.

3. Miss Wochner discussed a theorem of Sylvester's, proved a complementary theorem, and showed how they could be used to perform a Tschirnhaus transformation. A method was given for writing the adjoint of a matrix A, which is in standard form, from the characteristic equation of A. A method for computing high powers of a matrix was suggested.

4. Mr. Cowgill explained the theory of the harmonic analyser of Mader, and demonstrated its use. The curve $y=3\sin x+4\cos x+4\sin 2x-3\cos 2x-2\sin 3x-\cos 4x$ accurately plotted and then analysed, gave the equation $y=2.92\sin x+3.97\cos x+3.99\sin 2x-2.97\cos 2x-1.94\sin 3x-1.03\cos 4x$.

M. M. FLOOD, *Chairman*

HUNTINGTON'S THEOREM ON MOMENTS

By WILLIAM F. OSGOOD, Harvard University

Professor Huntington[1] appears to have been the first to obtain a necessary and sufficient condition for taking moments about a point, in the dynamics of a rigid body in two dimensions. By "taking moments about a point" is meant, writing the equation of moments in the form:

$$I\frac{d^2\theta}{dt^2}=\sum \text{moments}, \tag{1}$$

where I denotes the moment of inertia about the point, and the right-hand side of the equation is the sum of the moments of the forces about the point. His theorem is as follows.

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[1] *The theorem of rotation in elementary mechanics*, this Monthly, vol. 21 (1914), pp. 315–320.

Theorem. *It is permissible to take moments about a point Q distinct from the centre of gravity when and only when the vector acceleration of Q, thought of as a point fixed in the body, is collinear with the line joining Q with the centre of gravity, or vanishes.*

Huntington's proof is elegant and succinct, but it makes high demands on the maturity of the student. The following proof, though requiring more computation, is easier for the beginner to understand.

The equation of moments can be written in the form:

$$\frac{d}{dt}\sum m_i \left(x_i\frac{dy_i}{dt} - y_i\frac{dx_i}{dt}\right) = \sum \text{moments about the origin.} \tag{2}$$

It is the left-hand side only of this equation, with which we are concerned. Let the body be referred to moving axes, the point $Q{:}(x_0, y_0)$ being taken as the origin in that system. Then

$$\begin{cases} x = x_0 + \xi\cos\theta - \eta\sin\theta, \\ y = y_0 + \xi\sin\theta + \eta\cos\theta, \end{cases} \tag{3}$$

where x_0, y_0, θ are functions of t which determine the motion. Moreover,

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{dx_0}{dt} - (y - y_0)\dfrac{d\theta}{dt}, \\ \dfrac{dy}{dt} = \dfrac{dy_0}{dt} + (x - x_0)\dfrac{d\theta}{dt}. \end{cases} \tag{4}$$

Computing dx_i/dt and dy_i/dt from equations (4), we find:

$$x_i\frac{dy_i}{dt} - y_i\frac{dx_i}{dt} = x_i\frac{dy_0}{dt} - y_i\frac{dx_0}{dt} + \{x_i^2 + y_i^2 - x_0x_i - y_0y_i\}\frac{d\theta}{dt}.$$

Hence

$$\sum m_i\left(x_i\frac{dy_i}{dt} - y_i\frac{dx_i}{dt}\right) = M\left(\bar{x}\,\frac{dy_0}{dt} - \bar{y}\,\frac{dx_0}{dt}\right) + \{I - M\bar{x}x_0 - M\bar{y}y_0\}\frac{d\theta}{dt}, \tag{5}$$

where

$$I = M(k^2 + \bar{r}^2),$$

and k denotes the radius of gyration about the centre of gravity.

The most interesting case is that in which the point Q is taken as the instantaneous centre, (x_1, y_1), for the text-book literature is full of mistakes here, as Professor Huntington has pointed out. In this case, if we let Q be at the origin, O, of the (x, y)-system of coordinates at the arbitrary instant of time $t=t$, we shall have, at that instant,

$$x_1 = 0, \quad y_1 = 0, \quad \frac{dx_1}{dt} = 0, \quad \frac{dy_1}{dt} = 0, \tag{6}$$

but the second derivatives, d^2x_1/dt^2 and d^2y_1/dt^2 will not in general vanish.

On differentiating equation (5) with respect to the time and taking account of (6), we find that the left-hand side of (2) reduces to

$$I\frac{d^2\theta}{dt^2} + M\left(\bar{x}\frac{d^2y_1}{dt^2} - \bar{y}\frac{d^2x_1}{dt^2}\right).$$

It is this last term that must vanish, if we are to be allowed to "take moments about the instantaneous centre." It will, of course, vanish whenever the vector acceleration of Q vanishes; but otherwise only when

$$\bar{x}:\frac{d^2x_1}{dt^2} = \bar{y}:\frac{d^2y_1}{dt^2},$$

and this is precisely Professor Huntington's result.

The general case of an arbitrary point $Q:(x_0, y_0)$ can be treated in the same manner. It will be convenient to let Q flash through the origin, O, at an arbitrary instant, $t=t$. Here, $x_0=0$, $y_0=0$, but even the derivatives of the first order will not in general vanish. Still, the left-hand side of (2) reduces to

$$I\frac{d^2\theta}{dt^2} + M\left(\bar{x}\frac{d^2y_0}{dt^2} - \bar{y}\frac{d^2x_0}{dt^2}\right),$$

and Huntington's theorem is proved.

NEW MATHEMATICAL PERIODICALS

By R. C. ARCHIBALD, Brown University

There are probably few mathematicians who are aware that during the past five years at least 32 new mathematical periodicals have been started; they are listed below. Rumania heads the list with seven, followed by Italy with five and Poland with four. Argentina and the United States, each with three, come next; then Germany with two. Czechoslovakia, England, Holland, Hungary, India, Japan, Russia, Switzerland, each have one. Only four of these, the ones whose numbers are marked with a star, have not been inspected and are not in the library here. At least half of those listed may be regarded as research publications, namely, numbers 2, 3, 4, 10, 11, 13, 16, 20, 22, 23, 24, 26, 28, 30, 31, 32.

Rumania

*1. *Buletinul "Asociaţei matematice" a Liceului "C. Alimănestianu" din Oltenița*, Oltenița, v. 1, 1928–29.

2. *Bulletin de Mathématiques et de Physiques, Pures et Appliquées de l'Ecole Polytechnique de Bucarest*, Bucharest, v. 1, 1929–30, 238 p.
In French. Mainly articles in pure and applied mathematics.

3. *Bulletin Scientifique de l'École Polytechnique de Timişoara*. Comptes Rendus des Séances de la "Société Scientifique de Timişoara," Timişoara, v. 1, Dec. 1925–1928, 359 p.; v. 2, 1929, 255 p.; v. 3, 1930, 361 p.
There are many papers, practically all of them being in French.

4. *Mathematica*, Kolozsvár [Cluj], v. 1, 1930, 165 p.; v. 2, 1929, 183 p.; v. 4, 1930, 210 p.
Sixty-one articles, almost all of them in French, edited by a national committee.

*5. *Revista "Matematică Bârlădeană," Revistă lunară de Matematici Elementare*, Bârlad, v. 1, nos. 1–4, Jan.–Apr., 1926.

6. *Revista Universitară Matematică*, Bucharest, v. 1, 1929, 256 p.
Taking the place in Rumania that *Nouvelles Annales de Mathématiques* so long filled in France.

*7. *Ziarul Matematic. Revistă lunară pentru uzul Şcoalelar Secondare*, Chişinău, v. 1, nos. 1–5, Jan.–June, 1926.

Italy

*8. *Giornale di Matematica e Fisica della Scuola Media*, Reggio Calabria; there were, apparently, 2 nos. in 1927, 2 nos. in 1928, and 6 nos. in 1929.

9. *Rassegna di Matematica, Fisica e Scienze Naturali, Periodico mensile ad uso degli Studenti delle Scuole Medie*, Gioia dal Colle, v. 1, nos. 1–8, Nov. 1928–June 1929, 144 p.
No more published.

10. *Rendiconti del Seminario matematico della R. Università di Padova*, Padua, v. 1, 1930, 216 p.
Contains 10 articles by 8 authors.

11. *Rendiconti del Seminario Matematico e Fisico di Milano*, Milan, v. 1, 1927, 8+129 p.+2 plates; v. 2, 1928, 12+200 p.; v. 3, 1929, 16+267 p.
The contents include 39 articles.

12. *Rivista di Matematica Pura ed Applicata per gli Studenti delle Scuole Medie*, Palermo, v. 1, Dec. 1925–June 1926, 136 p.; v. 5 was completed in June, 1930. Paged consecutively 1–646, from v. 1, no. 1—v. 5, no. 8.

Poland

13. *Académie Polonaise des Lettres, Comptes Rendus Mensuels des Séances de la Classe des Sciences Mathématiques et Naturelles*, Cracow, v. 1, 1929, 78 p.; v. 2, 1930, nos. 1–7, 55 p.

14. *Mathesis Polska. Miesiecznik poświęcony Naukon Ścislym i ich Metodologji* [Polish Mathesis. Monthly devoted to the science of numbers and methodology], Warsaw, v. 1, 1926, 8+156 p.; v. 2, 1927, 8+136 p.; v. 3, 1928, 8+188 p.; v. 4, 1929, 8+186 p.; v. 5, 1930, nos. 1–8, 170 p.

15. *Parametr. Czasopismo poświecone Nauczaniu Matematyki* [Parameter. Polish periodical of mathematical science], Warsaw, v. 1, 1930, 400 p.
Devoted to the teaching of mathematics in elementary and secondary schools, 10 numbers a year.

16. *Studia Mathematica*, Lemberg, v. 1, 1929, 4+255 p.
Fourteen articles in French, German, and Italian, dealing with topics of functional equations.

Argentina

17. *Boletín del Seminario Matemático Argentino*, Buenos Aires, Sept. 1928–Sept. 1929, nos. 1–5; *Boletín de Seminario Matemático*, no. 6, Feb., 1930; *Universidad de Buenos Aires, Facultad de Ciencias Exactas Físicas y Naturales, Seminario Matemático*. nos. 7–9, March, 1930; 222 p.

18. *Boletin Matemático*, Buenos Aires, v. 1, 1928, 124 p.; v. 2, 1929, 192 p.; v. 3, 1930, 168 p.
Short articles, notes, and solutions of problems.

19. *Boletin Matemático Elemental*, Buenos Aires, v. 1, 1930, 36 p.

United States

20. *Annals of Mathematical Statistics*, American Statistical Association, Ann Arbor, Mich. v. 1, Feb.–Nov., 1930, 363+187 p.
Twenty articles, one with extensive tables.

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Dec., 1930.

SOME DETERMINANTS IN THE THEORY OF DEVELOPABLES

By R. P. BAKER, University of Iowa

If x, y, z, w are functions of t the theory of developables demands the consideration of the matrix

$$\begin{vmatrix} x, & y, & z, & w \\ x', & y', & z' & w' \\ x'', & y'', & z'', & w'' \\ x''', & y''', & z''', & w''' \end{vmatrix}.$$

The ratios of the elements of the first row give the points of the system; the ratios of the determinants formed from the first and second row (subject to the Plücker identity), the lines of the system; and the ratios of the determinants of the first three rows, the planes of the system. The determinant is the wronskian which vanishes only for a plane curve.

Now if X, Y, Z, W are the cofactors of x, y, z, w in the wronskian and we form the matrix

$$\begin{vmatrix} X, & Y, & Z, & W \\ X', & Y', & Z', & W' \\ X'', & Y'', & Z'', & W'' \end{vmatrix}$$

we expect and actually obtain the dual representation, namely: X, Y, Z, W give the planes; the first two rows, the lines in planar coordinates; and the third order determinants, the points of the system.

A repetition of the process should lead to a matrix whose first line ξ, η, ζ, ω is proportional to x, y, z, w but apparently ξ contains fourth derivatives, and the wronskian which we expect to occur as the proportionality factor contains only third derivatives. This seems to require explanation. We shall show that these extra derivatives are only apparent and that the theory can be extended to n functions.

1. As to the proportionality of ξ, η, ζ, ω to x, y, z, w we have, by the theory of determinants,

$$\sum xX = 0, \quad \sum x'X = 0, \quad \sum x''X = 0.$$

Differentiating, $\Sigma x'X + \Sigma xX' = 0$, $\Sigma x'X' + \Sigma x''X = 0$; hence $\Sigma xX' = 0$ and $\Sigma x'X' = 0$. Differentiating again, $\Sigma x'X' + \Sigma xX'' = 0$; hence $\Sigma xX'' = 0$. We have also $\Sigma \xi X = 0$, $\Sigma \xi X' = 0$, $\Sigma \xi X'' = 0$; and so

$$\xi : \eta : \zeta : \omega = x : y : z : w.$$

2. As to the proportionality factor, Y which appears in the first line of ξ is

$$\begin{vmatrix} x, & z, & w \\ x', & z', & w' \\ x'', & z'', & w'' \end{vmatrix}.$$

We may denote this by (012) from the fact that it contains derivatives of these orders.

Differentiating, $Y' = (013)$ and $Y'' = (014) + (023)$. The determinant for ξ can be represented as the sum of two determinants whose first columns are

$$\begin{matrix} (012) \\ (013) \\ (023) \end{matrix} \quad \text{and} \quad \begin{matrix} (012) \\ (013) \\ (014) \end{matrix} \quad \text{respectively.}$$

The first determinant is $-xV^2$ for the elements are minors of the wronskian V in reverse order of rows. We now have to prove that the second vanishes identically. Consider the determinant

$$\begin{vmatrix} x, & y, & z, & w, & 0 \\ x', & y', & z', & w', & 0 \\ x'', & y'', & z'', & w'', & 0 \\ 0, & y, & z, & w, & x \\ 0, & y', & z', & w', & x' \end{vmatrix} \equiv \begin{vmatrix} x, & 0, & 0, & 0, & -x \\ x', & 0, & 0, & 0, & -x' \\ x'', & y'', & z'', & w'', & 0 \\ 0, & y, & z, & w, & y \\ 0, & y', & z', & w', & x' \end{vmatrix} = 0.$$

Expanding by the first three rows,

$$\begin{vmatrix} x, & z, & w \\ x', & z', & w' \\ x'', & z'', & w'' \end{vmatrix} \begin{vmatrix} x, & y \\ x', & y' \end{vmatrix} + \begin{vmatrix} x, & y, & w \\ x', & y', & w' \\ x'', & y'', & w'' \end{vmatrix} \begin{vmatrix} z, & x \\ z', & x' \end{vmatrix} + \begin{vmatrix} x, & y, & z \\ x', & y', & z' \\ x'', & y'', & z'' \end{vmatrix} \begin{vmatrix} x, & w \\ x', & w' \end{vmatrix} = 0;$$

and we have a set of multipliers for the elements of the first row of ξ which gives a zero. Now the row of second derivatives may be replaced by any arbitrary quantities, for instance by the corresponding third or fourth derivatives, without affecting the result. Hence the determinant vanishes.

3. In the case of the determinants which should represent the lines in planar Plücker coordinates we have

$$\begin{vmatrix} Z, & W \\ Z', & W' \end{vmatrix} = \begin{vmatrix} [z'''], & [w'''] \\ [z''], & [w''] \end{vmatrix} = \begin{vmatrix} x, & y \\ x', & y' \end{vmatrix} V,$$

where $[z'']$ denotes the cofactor of z'' in the wronskian.

4. To generalize to n functions we study the first column of the determinant for ξ.

In short notation it is

$$(0, 1, 2, \cdots k),$$

$$(0, 1, 2, \cdots k-1, k+1),$$

$$(0, 1, 2, \cdots k-2, k, k+1) + (0, 1, 2 \cdots k-1, k+2),$$

$$(0, 1, 2, \cdots k-3, k-1, k, k+1) + 2(0, 1, \cdots k-2, k, k+2) + (0, 1, \cdots k-1, k+3),$$

. . . . . .

. . . . . .

$$(0, 2, 3, \cdots, k+1) + \binom{k-1}{1}(0, 1, 3, \cdots, k+1) + \binom{k-1}{2}(0, 1, 2, 4, \cdots).$$

The binomial coefficients are immaterial for our purpose. We notice that the first column contains minors of the wronskian (in reverse order). They lack in turn the symbol $k+1, k \cdots 3, 2, 1$. The last one is the only one without a

1, the penultimate the only one without a 2, and so on. An aggregate selected one from each row will have a pair of common symbols. If not (01), we must use the wronskian minor in the last row. If we select this and if (02) is not a common pair we must take the preceding wronskian minor. Ultimately we have all the wronskian minors or a common pair of symbols in the aggregate.

The property used in the special case $k=2$ was that an aggregate with a pair of common rows formed with the other columns similarly represented formed a vanishing determinant. This obviously generalizes.

The final result is: $\xi_1=(-1)^{n-1}\ V^{n-2}$, where n is the number of functions.

For the minors of the third and higher orders the multiplier method must be used to rid the aggregate of determinants containing the derivatives of order n and higher.

5. In these proceedings differentiation has apparently been replaced by a formal symbolic process. The properties actually used are $d(u+v)=du+dv$ and $d(uv)=udv+vdu$; the fact that $dt/dt=1$ was not appealed to. This allows us apparently to generalize the operator to either $\lambda\cdot dx/dt$ or $d(\lambda x)/dt$ where λ is an arbitrary function of t. However in case we do the geometric interpretation depending only on ratios of determinants is unchanged.

A NOTE ON INSTRUCTION IN MECHANICS

By ALEXANDER WUNDHEILER, University of Warsaw

Certain common methods of solution of problems in mechanics contain a source of error. We shall show that if we make use of the energy integral, impossible motions are generally obtained.

The common method of treatment of holonomic mechanical systems, the constraints and the potential energy not involving the time, consists in first writing down the energy integral. This is sufficient, if the system has only one degree of freedom. If not, we then seek other equations, e.g. the Lagrangian ones. It will be shown by some examples that the system containing the energy integral and other equations admits, generally, solutions alien to the given, not integrated, system.

Consider the simple harmonic motion of a particle attracted toward a center proportionally to the distance. The potential function is $\frac{1}{2}mk^2x^2$ and the energy integral is

$$\dot{x}^2 = k(a^2 - x^2), \tag{1}$$

where a is a constant. If we seek the motion corresponding to the initial conditions $t=t_0$, $x=a\neq 0$ we find *two* solutions. The one is $x=\text{const.}=a$, and signifies equilibrium at the point a. The second one is given by the quadrature:

$$t - t_0 = \int_{x_0}^{x} \frac{dx}{k^{1/2}(a^2 - x^2)^{1/2}} = k^{-1/2} \arcsin \frac{x - x_0}{a}.$$

It is here obvious that the first one is wrong, and we prove it by verifying that the differential equation of second order of the motion,

$$\ddot{x} = -kx, \tag{2}$$

does not admit the solution $x=a$. In the present case, the falseness of the first solution is evident (for $a \neq 0$), and there is no difficulty in finding out where the alien solution comes in. In fact, in deducing the energy integral, we multiply (2) by $\dot{x}$, and introduce, therefore, the solution of the equation $\dot{x}=0$, viz. $x=a$.

We shall now consider cases where the falseness of the alien solutions is not so evident, and where their explanation is not so simple. Consider Kepler motion, that is, the central motion of a particle attracted toward a center proportionally to the inverse square of the distance. The energy integral in polar coordinates is

$$\dot{r}^2 + r^2\dot{\phi}^2 = 2\mu r^{-1} + h, \tag{3}$$

and the integral of areas is

$$r^2\dot{\phi} = C. \tag{4}$$

It is well known that all circumferences, the centers of which are the center of attraction, are possible trajectories. We ask if the manner in which the particle describes them is uniquely determined. Putting in the equations (3) and (4) $r=r_0$, we obtain

$$r_0^2\dot{\phi}^2 = 2\mu r_0^{-1} + h, \quad \dot{\phi} = Cr_0^{-2}.$$

As h and C are arbitrary constants we infer that $\dot{\phi}$ may have, along such a circumference, an arbitrary but constant value, i.e. that such a circumference may be described by a particle with an arbitrary velocity. Now, this is false. In fact, the intrinsic equation for the normal acceleration, which in the present case takes the form

$$mv^2r_0^{-1} = m\mu r_0^{-2}$$

shows that the velocity is completely determined by the normal component of the force and the radius of curvature of the trajectory. We obtain

$$V = (\mu r_0^{-1})^{1/2}$$

and the other values of v must be considered as alien solutions, introduced when effecting the computation leading to the energy integral. But the indication of the precise moment where this introduction took place is not quite so simple as in the former example.

We find another quite analogous case in the theory of the spherical pendulum. The reader will verify immediately, after having written down the energy

integral and the integral of areas, that they give for all parallels of the sphere an arbitrary velocity, whereas the intrinsic equations of the motion give a completely determined velocity.

The generalization of these examples does not present any difficulty. But it is easily seen that in the more complicated problems concerning systems of material particles and rigid bodies, our intuitive knowledge of the motion will not suffice to guess which are the alien motions. Consider, for instance, a homogeneous, rigid, heavy rod, one extremity of which slides without friction along a vertical line, and the other one on a horizontal plane. Let the angle of the rod with the vertical be θ. If we examine the motion by means of the integrals of energy and areas, we obtain the result that the stationary motion, $\theta=$ const., is possible in connection with an arbitrary value of $\dot{\phi}$ (ϕ designating the angle of the horizontal projection of the rod with a fixed horizontal direction). It would seem that in the present case it is not quite easy to guess that corresponding to a given value of θ, only one determined value of $\dot{\phi}$ is possible, as is seen when we make use of the Lagrangian equations of the motion.

THE SOLUTIONS OF $x^y=y^x$, $x>0$, $y>0$, $x\neq y$, AND THEIR GRAPHICAL REPRESENTATION

By H. L. SLOBIN, University of New Hampshire

The equation

$$x^y = y^x, \tag{1}$$

is equivalent to

$$y \log x = x \log y \quad \text{or} \quad \frac{\log x}{x} = \frac{\log y}{y}.$$

For convenience we shall write this equation in the form

$$\frac{\log x_1}{x_1} = \frac{\log x_2}{x_2}. \tag{2}$$

If now we consider the graphs of $y=\log x$ and $y=mx$ we note that $y=0$, the x-axis, is parallel to the curve, $y=\log x$, as x becomes infinite, since $dy/dx=1/x$ when $y=\log x$. We may now determine the value of m for which $y=\log x$ and $y=mx$ meet and have the same slope. Since $\log x=mx$ and $m=1/x$ we have $\log x=1$ or $x=e$ (the base of the system of natural logarithms).

It is thus evident that the lines $y=mx$ for $0<m<1/e$ will each have two intersections with $y=\log x$. For each such line we thus determine a pair of numbers x_1 and x_2, the abscissas of the points of intersection of $y=mx$ and $y=\log x$, so that

$$\frac{\log x_1}{x_1} = \frac{\log x_2}{x_2}.$$

To determine the solutions of $x^y=y^x$, where both x and y are rational numbers, is merely to determine the pairs of rational numbers x_1 and x_2 satisfying equation (2).

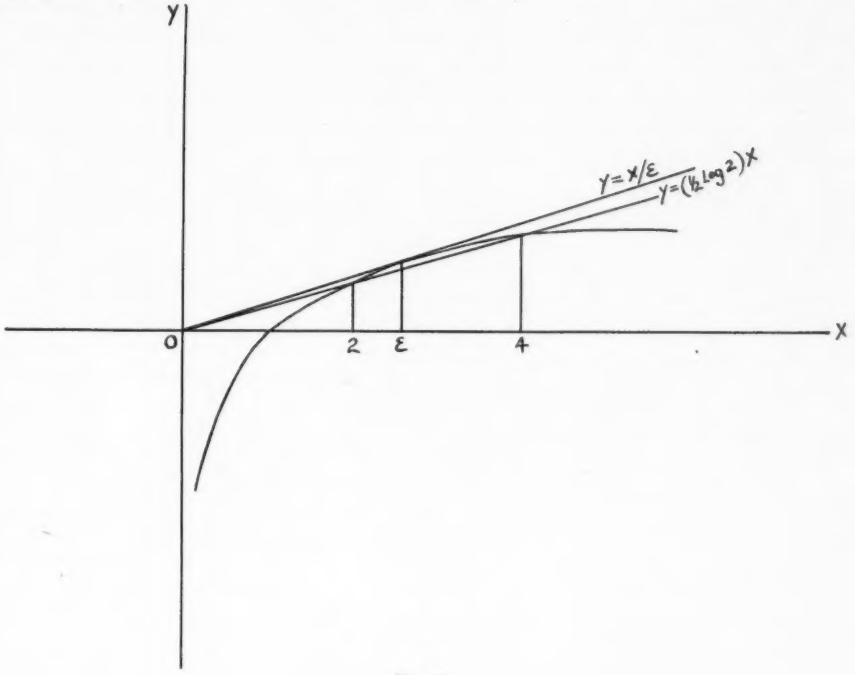

FIG. 1.

If we assume x_1 to be a rational number, then $x_2=x_1r$, where r is undetermined but must be rational if x_2 is to be rational. Hence

$$\frac{\log x_1}{x_1} = \frac{\log (x_1r)}{x_1r} \quad \text{or} \quad \left(1-\frac{1}{r}\right)\log x_1 = \frac{\log r}{r},$$

or

$$x_1 = r^{1/(r-1)}, \qquad x_2 = r^{r/(r-1)}.$$

If we assume $x_2>x_1$, then $r>1$. Therefore letting $r=1+s$, we have

$$x_1 = (1+s)^{1/s}, \text{ and } x_2 = (1+s)^{(1+s)/s}. \tag{3}$$

If we let

$$s = 1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n},$$

we have

$$x_1 = 2, \frac{9}{4}, \frac{64}{27}, \cdots, \left(1+\frac{1}{n}\right)^n \cdots,$$

and

$$x_2 = 4, \frac{27}{8}, \frac{256}{81}, \cdots, \left(1+\frac{1}{n}\right)^{n+1} \cdots.$$

It will be noted that as s was chosen to traverse a set of rational numbers from 1 to 0, x_1 traversed a set from 2 to e, and x_2 traversed a set from 4 to e.

In seeking *all* the pairs of rational numbers x_1 and x_2 that satisfy equations (3), we must confine ourselves to s rational. Hence letting $s=p/q$, where p and q are prime to each other, we have

$$x_1 = \left(\frac{p+q}{q}\right)^{q/p}, \quad x_2 = \left(\frac{p+q}{q}\right)^{(q+p)/p}. \tag{4}$$

Since x_1 and x_2 are to be rational numbers, $(p+q)^{1/p}$ and $q^{1/p}$ must be integers, say n_1 and n_2, respectively. Hence

$$x_1 = \left(\frac{n_1}{n_2}\right)^q \text{ and } x_2 = \left(\frac{n_1}{n_2}\right)^{p+q}. \tag{5}$$

Letting $n_1-n_2=t$, we have

$$(n_2+t)^p = n_1{}^p = p+q, \qquad n_2{}^p = q.$$

Hence

$$(n_2+t)^p - n_2{}^p = p. \tag{6}$$

Expanding (6) we have

$$pn_2{}^{p-1}t + \frac{p(p-1)}{2!}n_2{}^{p-2}t^2 + \cdots + t^p = p. \tag{7}$$

Obviously equation (7) cannot be satisfied if n_2 and t are integers. Hence the assumption $s=p/q$ does not give rational values for x_1 and x_2, unless $p=1$ or $q=1$.

If $p=1$, $s=1/q$ and

$$x_1 = \left(\frac{1+q}{q}\right)^q, \text{ and } x_2 = \left(\frac{1+q}{q}\right)^{q+1}. \tag{8}$$

We can now let $q=1, 2, 3$, and then have from (8):

$$x_1 = 2, \frac{9}{4}, \frac{64}{27}, \cdots, \quad x_2 = 4, \frac{27}{8}, \frac{256}{81}, \cdots,$$

which is the aggregate already determined.

If $q=1$, $s=p$ and hence

$$x_1 = (1 + p)^{1/p}, \text{ and } x_2 = (1 + p)^{(1+p)/p}. \tag{9}$$

In this case x_1 and x_2 cannot be rational unless $(1+p)^{1/p}$ is an integer. But it is evident from the graph $y = \log x$ that the value of x_1 must lie between 1 and e, and the *only* possible integer is 2. This value for x_1 corresponds to $p = 1$ and gives $x_1 = 2$ and $x_2 = 4$, which has already been obtained.

The significance of our conclusions is interesting if interpreted graphically. Each of the lines $y = mx$, where $\frac{1}{2}\log 2 \leqq m \leqq e^{-1}$, cuts the curve in two points, and the points whose abscissas are both rational numbers have been determined; but if $0 < m < \frac{1}{2}\log 2$, the abscissas of the points of intersection cannot *both* be rational.

It is also evident that we cannot have two intersections for each value of m save for $0 < m < e^{-1}$.

The problem is capable of expansion to similar problems. Thus the equation

$$(e^x)^{(e^y)} = (e^y)^{(e^x)}$$

cannot have a single pair of values x and y both algebraic numbers (that is rational numbers and algebraic irrational numbers). This is evident since we have $xe^y = ye^x$ or $x/y = e^{x-y}$. If x and y were both algebraic numbers x/y and $x-y$ would be algebraic numbers while e^{x-y} would be a transcendental number. (See the author's paper in the Rendiconti di Circolo Mathematico di Palermo, vol. 38 (1914), p. 353.)

QUESTIONS AND DISCUSSIONS

EDITED by R. E. GILMAN, Brown University, Providence, Rhode Island.

The department of Questions and Discussions in the Monthly is open to all forms of activity in collegiate mathematics, including the teaching of mathematics, except for specific problems, especially new problems, which are reserved for the department of Problems and Solutions.

A THEOREM ON FOCI

By EDWIN J. PURCELL, University of Colorado

THEOREM: *If a real plane algebraic curve of class m has one or more real axes of symmetry, then in general exactly m foci lie on each axis of symmetry; any such set of m collinear foci completely determines the remaining foci.*

Proof: Consider any real line of symmetry of the curve. Let this line be the x-axis in rectangular coördinates. The tangential equation of the curve will have real coefficients and be of the form

$$f(u, v^2) = 0. \tag{1}$$

Letting $v = \pm i/c$ in (1), there will be in general exactly m values of u satisfying (1). Interpreted geometrically, this means that there will be exactly m intersections of isotropic tangents lying on the axis of symmetry. These m in-

tersections are m foci (real or imaginary) of the curve. Since the axis of symmetry is a real line, not the line at infinity, it does not pass through I or J, the circular points at infinity. The m foci on it completely determine the m tangents to the curve from I and the m tangents to the curve from J. The other intersections of these 2 m tangents are the remaining (m^2-m) foci. The theorem follows.

If k foci on an axis of symmetry coincide, then the number of foci on that axis is increased by (k^2-k), since such a focus counts k^2 times. If the curve is tangent to the line at infinity so that the number of real (finite) foci is reduced from m to p, say, then exactly p foci will lie on each axis of symmetry.

The Characteristic Equations of the Adjoint and the Inverse of a Matrix

By H. S. Thurston, University of Alabama

The following theorem may possibly be found somewhere in current mathematical literature, but the writer is unaware of the previous publication of a proof.[1]

Theorem. If a is a non-singular matrix of order n and determinant D, a^{-1} its inverse and A its adjoint, then the roots of the characteristic equation of A are respectively D times the roots of the characteristic equation of a^{-1}, and the latter are the reciprocals of the roots of the characteristic equation of a.

The characteristic functions of a, A, and a^{-1} are respectively

$$\phi(\lambda) \equiv s_n - s_{n-1}\lambda + s_{n-2}\lambda^2 - \cdots + (-1)^n\lambda^n$$

$$\Phi(\lambda) \equiv S_n - S_{n-1}\lambda + S_{n-2}\lambda^2 - \cdots + (-1)^n\lambda^n$$

$$\psi(\lambda) \equiv p_n - p_{n-1}\lambda + p_{n-2}\lambda^2 - \cdots + (-1)^n\lambda^n$$

where s_k, S_k, and p_k are respectively the sums of all k-rowed principal minors of a, A, and a^{-1}.

On page 31 of Bôcher's *Introduction to Higher Algebra*, we find the following theorem: If D' is the adjoint of any determinant D, and M and M' are corresponding m-rowed minors of D and D' respectively, then M' is equal to the product of D^{m-1} by the algebraic complement of M. Although the adjoint of a matrix and the adjoint of a determinant are in form conjugate to each other, the k-rowed principal minors of both have the same value, and from the above theorem we see that

$$S_{n-k} = D^{n-k-1}s_k.$$

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[1] For the theorem on the reciprocal see Pascal's *Repertorium*, Vol. I Part I, page 117 *Editor.*

Since each element of A is D times the corresponding element of a^{-1}, it follows at once that

$$p_{n-k} = \frac{S_{n-k}}{D^{n-k}} = \frac{s_k}{D}.$$

From the relations $S_{n-k} = D^{n-k} p_{n-k}$ and $s_k = D p_{n-k}$, it is clear by elementary theory of equations that the roots of $\Phi(\lambda) = 0$ are D times the roots of $\psi(\lambda) = 0$, while those of $\psi(\lambda) = 0$ are the reciprocals of those of $\phi(\lambda) = 0$.

If a is singular, of rank r, the theorem becomes trivial even in a modified form. The rank of A is 1 or 0 according as $r = n-1$ or $r \leqq n-2$. If $r = n-1$, $\Phi(\lambda) = 0$ has one root different from 0; if $r \leqq n-2$, all roots of $\Phi(\lambda) = 0$ are 0.

A New Method for Solving the Equation $x^x = c$

By E. C. Kennedy, University of Texas College of Mines

Equations of the type $x^x = c$ may be solved by finding two approximate values of x and getting a better value by straight line interpolation. If the process is repeated several times an accurate result may be obtained.

A far quicker, easier, more accurate method involving less chance of error is described below.

Let $x^x = (P_0 + h)^{(P_0+h)} = c$, where P_0 is a rough approximation to the value of x. Then $P_0^{(P_0+h)}(1+h) = c$, approximately. Taking natural logs of both sides and assuming $\log(1+h) = h$ we obtain for our first approximation, P_1,

$$P_1 = (P_0 + h) = (P_0 + \log c)/(1 + \log P_0).$$

To get a second approximation, P_2, we write

$$P_2 = (P_1 + h') = (P_1 + \log c)/(1 + \log P_1).$$

The nth approximation is readily seen to be

$$P_n = (P_{n-1} + \log c)/(1 + \log P_{n-1}).$$

To illustrate the brevity of this method we shall solve the equation $x^x = 32.46$. Taking $P_0 = 3$ (any value in this neighborhood will do) we have

$$P_1 = (3 + \log 32.46)/(1 + \log 3) = 6.480/2.099 = 3.087.$$

Thus $x = 3.087$ is the first approximation. (Newton's method gives 3.096). In order to get the maximum degree of accuracy obtainable by a second approximation it is generally necessary to use a 7-place table. Thus

$$P_2 = \frac{3.087 + \log 32.46}{1 + \log 3.087} = \frac{6.5670085}{2.1271997} = 3.0871613,$$

a result correct to at least 7 places. A third trial takes our answer to about 12

places. After the first approximation the accuracy of the result increases amazingly.

By this method it is almost impossible to go wrong for even if an inexcusably poor value of P_0 is taken the error will automatically eliminate itself with one or two more approximations. For example, if $c=100$ we should choose $P_0=$about 3.4 or 3.5 or 3.6 instead of 3, although the latter will serve of course. For two approximations (and seldom more than two are needed) no multiplications and only two divisions are required.

If $c<.692\cdots$ there is no positive solution in real numbers. If $.692\cdots<c<1$, then there are two solutions—at least two greater than zero. To get one of these choose P_0 slightly greater than $1/\epsilon$, to get the second choose P_0 slightly less than $1/\epsilon$ and the rest is purely mechanical.

This method is applicable to equations of the form $u^u=c$, where $u=u(x)$. For example,

$$\log (x-5)^{\log(x-5)} = 7.2, \quad \tan (x-.4)^{\tan(x-.4)} = 9.68, \text{ etc.}$$

may be solved very accurately in four or five minutes. If one were to solve ten or twelve equations of this type it would be best to plot $y=x^x$ carefully, then by making use of the graph and one approximation the root could be found to 5 or 6 places in two minutes.

Newton's method could be used on such equations, but the method described above is much superior. In fact, it does not seem possible that there could be an easier way of solving such equations than that given.

RECENT PUBLICATIONS

EDITED by ROGER A. JOHNSON, Brooklyn College of the City of New York

All books for review should be sent directly to the editor of this department, at Brooklyn College, 66 Court Street, Brooklyn, N. Y., and not to any of the other editors or officers of the Association.

REVIEWS

Intergralgleichungen. By G. Kowalewski. Berlin, Walter de Gruyter & Co., 1930. 302 pages.

This book is one of "Göschen's Lehrbücherei," and is not to be regarded as a complete treatise. Rather, it seems designed to occupy the position in German that the several times reprinted "Tract" of Bôcher still fills so well in English. The exposition of Kowalewski is exceedingly detailed, perhaps unnecessarily so, but also, on that account, very easy reading; it is largely algebraic, and the subject matter is governed by the traditional choice. The chapter of applications at the end, however, is hardly sufficient to give the reader any hold, and the "Literaturhinweise" is scanty. A good many special integral equations are

worked through in complete detail, rather as applications or illustrations of the general theory than as indications of possibilities that are not covered by it.

Hence although there are some indications of a group theory and of a theory of functional transformations, the reader is not led into new problems and unsolved questions. He is likely to regard the subject as a circle in which all the roads run parallel to the circumference. The book is an able exposition, and will serve to introduce the student to the subject. But the reader, having finished it, should turn elsewhere, say to Carlemann, in order to be shocked into renewed vitality.

The introduction deals with Abel's equation and other special inversions of integral formulae, like the Fourier integrals, considered as functional transformations; Chapter I is devoted to Volterra integral equations, with a brief analysis of the algebraic or symbolic properties of the integral operator; Chapter II treats similarly the Fredholm type, with incidental reference to elementary divisors; Chapter III gives the theory of symmetric kernels; and Chapter IV discusses briefly two applications—the vibration problem for a string, and, without much proof, the Dirichlet problem for a region interior to a closed curve.

GRIFFITH C. EVANS

Tutorial Exercises in Trigonometry. By Raymond W. Brink and Ella Thorp. Century Company, New York, 1931. 102 pages, 9×11. Paper bound. $1.25.

The need for carefully selected and well-arranged exercises in trigonometry has been realized in the publication of this book. In each set of exercises not only is the student led by definite, graded steps to the more difficult problems, but enough exercises are given to effect a drill on each principle. Then too ten sets of review exercises, besides a final, all-embracing test, serve to organize the work of the preceding lessons.

The form of the book suits its purpose. Each sheet, printed on only one side, presents a neat appearance. Exercises, diagrams, and room for working have been carefully spaced. Furthermore the heading of each page by a concise statement of the topic covered is an aid to both teacher and student. The book should prove interesting and helpful as a supplement to any text in trigonometry.

HARRIET GRIFFIN

Il Passato e il Presente delle Principali Teorie Geometriche. Storia e bibliografia. Fourth edition. By Gino Loria. Padua, Cedam, 1931. xxiii+467 pp. Lire 60.

In this interesting volume, the author presents, in cursive style, an outline of the principle theories of geometry, with emphasis on their historical development. He states briefly the chief results that have been arrived at in each of the various fields of geometry and gives numerous references for the benefit of readers who may wish to pursue any topic further. The work is one for which the author is conspicuously well fitted and to which he brings a wealth of informa-

tion and critical judgment. In spite of unfortunately numerous typographical errors, the book is decidedly well written. For anyone who wishes to take a hasty but interesting trip through the domain of geometry, to have pointed out to him the main topics that have been developed, with their modes of treatment and the outstanding results, this volume will serve as an excellent guidebook.

The present fourth edition is based on the second, which was published in 1896. The first 291 pages constitute a thorough revision of that earlier edition. The remainder of the text traces, in similar fashion, the development of geometry since 1896.

C. H. SISAM

Plane and Spherical Trigonometry. By R. D. Carmichael and E. R. Smith. Boston, Ginn & Company, 1930. xii+198 pages. $1.60.

"There is very little room for novelty in preparing a textbook for an introductory course in trigonometry. The material and methods are fairly well standardized."

It is scarcely to be expected that a trigonometry which carries the two sentences quoted above as the opening statements of its preface will incorporate any theoretical innovations of development or choice of material. The reader may rest assured at once that this textbook contains nothing which has not been fully tested in the class room. The authors have had long and successful experience in both writing and teaching and their book is conservative, teachable, well-written and mathematically sound.

The selection and arrangement of subject-matter is such as would probably be approved as first choice by a majority of those teaching the subject. Measurement of angles, definitions of the trigonometric functions for positive acute angles, elementary identities, general definitions of the functions, solutions of right triangles by means of natural functions, solutions of oblique triangles by means of right triangles, reduction formulas and graphs, addition formulas, solutions of triangles in general, DeMoivre's theorem and related identities, spherical triangles and their applications—this organization and development of material would generally be conceded to be both logical and psychological.

Moreover, the general treatment is such as to permit of considerable flexibility and adaptation to different types of classes and to teachers' preferences. If a very short course is desired, the first three chapters (pp. 1–66) form a complete unit covering the most essential ideas of the subject. If an instructor would prefer to omit the definitions of the functions for acute angles and begin at once with the definitions for general angles, that can easily be done. If he prefers to omit or abbreviate the solutions of right triangles by means of natural functions or the solutions of oblique triangles by means of right triangles, trusting to the later more complete treatment of triangles in Chapter VII, that can be done; if time presses, a number of starred supplementary sections can also be omitted without destroying the continuity of the course

Occasionally statements appear which lead to queries but scarcely to definite criticisms. On page 3 we read "A *straight line* . . . extends indefinitely in two directions. A *straight line segment* is that part of a straight line which lies between any two points on the line. A *ray*, or *half-line*, is that part of a straight line which lies on either side of a point on the line," and on page 4, ". . . a ray has direction, one end-point, but not length. A line segment is generally referred to as a line when the context admits of no doubt as to the meaning." Is it objectionable to say that a ray is longer than any segment of it or that the length of a ray is greater than that of any segment of it, even though the context might admit of no doubt as to the meaning? Would such usage of terms be any more objectionable than to speak of a line segment, which certainly has length, as a line, which certainly does not have length (in the same sense that a ray does not have length) under like conditions?

On page 15, line 1, we find "If two variable quantities, x and y, are so related . . . ." Why not simply "If two variables, x and y, are so related . . . ," since, immediately, in the first line of the next paragraph, the authors say" The variable x is called the independent variable and y the dependent variable or function of x?" Also, on page 43 we find "Such numbers as $\sqrt{2}, \frac{2}{3}, \cdots$, constants like π . . . and e . . . and many other quantities . . . ." Are $\sqrt{2}$, $\frac{2}{3}$, π and e quantities or definitely and precisely numbers which are sometimes used to state a measure of quantity? Possibly both, but the writer believes that there is a growing tendency not to use the indefinite (and, as sometimes used, obscure) term "quantity" indifferently for all sorts of concepts (number, variable, function, etc.) but to use the definite and precise term most appropriate whenever possible.

Again, on page 22, the student is asked to "check" certain identities by substituting 30° for the angle and finding the numerical values of the members. While "check" is often used in this limited sense, should not such use be discouraged, because of the danger that students may come to believe that substitution of a single numerical value forms a complete check? The exercise can be stated readily in terms which carry no such implication.

No material errors, however, were noticed, exercises are adequate and well graded and special attention seems to have been given to the arrangement of numerical work throughout the book.

It is hardly necessary to add, except by way of compliment, that the printer's part has been very well done. The imprint of the Athenaeum Press on a mathematical book is assurance of quality.

U. G. MITCHELL

Advanced Mathematics for Students of Engineering and Physics, I and II. By D. Humphrey. Oxford University Press, London, 1929. Part I, iv+120 pages, and Part II, iv+175 pages, bound in a single volume.

In this book the author aims to give those students of engineering and physics who have completed elementary calculus a knowledge of and training

for manipulative skill in as many branches of mathematics required in their other courses as he can present in a book of moderate size. He considers that most books on practical mathematics obscure and limit the mathematics involved by devoting too much space to the working out of practical problems and that manipulative skills in pure mathematics should be acquired before applications are attempted. The book, based on the work done at the Polytechnic, London, has been designed in accordance with these views.

The text is divided into seventeen chapters. Two chapters, containing forty-four pages, are on integration; five, containing eighty-eight pages, on differential equations; the remaining ten, containing one hundred forty-one pages, on series, complex quantities, applications to geometry, solid geometry, calculus of finite differences, determination of laws, numerical solution of equations, determinants, Fourier series and harmonic analysis, and vectors. The four hundred or so exercises, which are accompanied by answers, are almost entirely of an abstract nature; the few which are not deal only with very simple applications.

In the selection and arrangement of his material the author has not attempted to please everyone nor has he gone out of his way to make it particularly attractive to those students who require constant motivation in their courses. Probably everyone will agree that the emphasis on differential equations is rightly placed. Since each reader has his own tastes and feels his own special needs most, the amount of space allotted to the other topics is open to discussion of such length that it will not be considered here. On the whole, the reviewer finds the shorter topics, as well as the longer ones, clearly, directly, and accurately presented and, he believes, with a degree of rigor and completeness corresponding to the academic status of the student for whom they are intended. He feels, however, that the insertion of some discussion of the validity of infinite processes would make them more clear to the student and that alterations in two or three objectionable statements, of which the most important is the statement that the differential of $f(x)$ is the *actual* change when x is increased by dx, would contribute accuracy well within the student's powers of appreciation. The reviewer now wishes to prove his worth as such by adding to the above minor criticisms his support of the author in the matter of motivating material. There is a strong movement afoot to increase the amount of secondary material included in a course for the purpose of linking it up with other subjects to which it may apply. So strong has this tendency become that there is some danger of its being overdone. Indeed, accusations have been made that some text-books and some courses consist entirely of so-called motivating material to the exclusion of tangible principles on which to base the work. Such material is undoubtedly of great value in numerous connections, but the wisdom of using it beyond the first or second college year in well organized professional courses, such as engineering, is questionable. The engineer who after his first two years of training has to be further wheedled into preparing for the profession ought to be encouraged to try something else.

The reviewer recommends the book for closer examination by instructors who offer courses including the topics mentioned and who desire little motivating material in their texts and by independent students who desire a knowledge of and manipulative skill in these topics. It is well worthy of consideration as a mathematical contribution to the literature of engineering.

EARL L. MICKELSON

The Volterra Integral Equation of Second Kind. By Harold Thayer Davis. Indiana University Studies Nos. 88, 89, 90. 1930. 71 pages. Price $1.00 in paper cover, $1.25 in cloth.

This is the third of a series of monographs by the author on the general field of integral equations. The subject of discussion is the equation

$$u(x) = f(x) + \lambda \int_a^x K(x, t)u(t)dt$$

in the unknown function $u(x)$.

In Part I the classical method of successive substitutions is shown to lead to Volterra's resolvent kernel, and the case where $K(x, t)$ is a function of $(x-t)$ is examined by means of functions permutable with unity. Differential and integral operators of infinite order are introduced and are used to solve the equation when $K(x, t)$ is expandible as

$$\sum_{n=0}^{\infty} a_n(x)(x - t)^{n-1}/(n - 1)!.$$

Part I closes with a discussion of the case $K(x, t) = g(x, t)/(x-t)^{\alpha}$, $0 < \alpha < 1$, using fractional differentiation and integration.

Part II considers the singular equation, with either the constant limit infinite, or the kernel having an infinite discontinuity of type other than considered at the close of Part I. Results of Evans and Love are set forth and extended, the theorems covering several cases where there exist families of solutions.

The author has inserted several illustrative examples and problems, the latter containing just sufficient difficulty to whet the reader's appetite, without destroying it. The list of references is numbered to correspond with the complete bibliography in the first of this series,[1] and includes six entries added since that was published. A portrait of Vito Volterra makes an appropriate frontispiece, and adds to the appearance of the book.

The work is hardly to be recommended to the beginner, for, although no previous knowledge of integral equations is presumed, the large number of ideas introduced would tend to overwhelm him. However, it should carry a person with some familiarity with the subject and a good grounding in analysis, up to the frontier where research is being carried on.

L. S. KENNISON

[1] *The Present Status of Integral Equations*, Indiana University Studies, No. 70 (1926).

Types of Mathematical Thinking. Von Zahlen und Figuren. Proben mathematischen Denkens, für Liebhaber der Mathematik. Ausgewählt und dargestellt von Hans Rademacher und Otto Toeplitz. Julius Springer, Berlin, 1930; v+164 pages, 129 figures.

The purpose of this book is, as indicated in the sub-title, to give to the layman, through a number of examples of typical mathematical reasoning, a feeling for the real character of mathematical science. A brief table of contents will give some idea of the range of problems which the authors have been able to bring within the range of readers possessing no technical knowledge whatever beyond a few of the most important of the theorems of plane geometry and the rudiments of algebra. Here it is:

On the series of prime numbers. The continuous tracing of a net of curves (each branch once and only once). Some minimal properties (rectangles with given areas, inscribed triangles, inscribed polygons). Incommensurable, segments and irrational numbers. A minimal property of the triangle joining the feet of the altitudes of a given triangle, after H. A. Schwarz. The same property, after L. Fejér. Some points in the theory of aggregates. Plane sections of the right circular cone. The Waring problem. On closed knotted curves. Is the decomposition of a number into its prime factors unique? The four color problem. The regular polyhedra. Pythagorean numbers, and a glimpse of the Fermat problem. The Pferch circle of a given (finite) set of points—i.e. the circle with least radius enclosing the set. Approximation of irrational numbers by rationals. The drawing of straight lines by linkages. Perfect numbers. Euler's proof that the sequence of primes is infinite. Fundamental remark on maximum problems (warning against assumption of existence of the maximum, with illustration). Figure of greatest area with given perimeter (Steiner's linkage method). Periodic decimals. A characteristic property of the circle. Curves of constant breadth. Indispensibleness of the compass in the constructions of elementary geometry. A property of the number 30 (namely, that it is the greatest number such that all numbers below it with which it has no common factors are prime numbers).

This indication of contents brings to light a most varied group of topics, many of them classic, some involving still unanswered questions, some quite modern. It cannot indicate the grace with which they are treated, the simplicity, the naturalness, the interesting historical introductions, or the ever alert playing up of viewpoints or methods which, arising naturally in the problems, are characteristic of mathematical thinking. Difficulties of proofs are reduced to a minimum by concrete introductions and by judiciously selected preliminary lemmas. But rigorous methods are used in all cases where proofs are given, for otherwise the purpose of the undertaking would be vitiated.

The book should find a wide distribution among laymen, for the number of persons who are curious as to the nature of mathematical thought, and who are capable of understanding it, is constantly increasing. Probably there are many readers of the MONTHLY to whom questions have been addressed which this

book is well suited to answer. But it will also prove of great interest to the mathematician himself, because of the insight it gives into a variety of problems and points of view, at least some of which are likely to have novelty for any given person, because its style is a model of clarity and simplicity for those engaged in instruction, and because it is a mine of material for anyone called upon for talks, either to laymen or to students' mathematical clubs.

The authors, eminent as productive scientists, have here engaged in a work of popularization on a most dignified plane. They have not been concerned with puzzles or recreations, nor with persuading a reluctant public of the value of mathematics. They have given a genuine glimpse of the science itself, in a number of its phases, free from technical intricacies and symbolism. Without doubt the book deserves a place in every mathematical library. A translation into English, making the book available to still wider circles of laymen in this country, would be of great value.

O. D. KELLOGG

Éléments de Mathématiques Financières. By R. Thiry. Paris, Librairie Vuibert 1930. viii+87 pages.

The content of the book consists of material which the author has presented frequently as a course at l'Institut d'Enseignement Commercial Supérieur de Strasbourg. This course is one for the *general* students of commerce, and not primarily for those who will specialize in the mathematics of investment. The author's object was to present in this book the minimum essentials of the theory of the subject, together with a few of its most important applications. It is interesting to note that the author, in common with many persons in other parts of the world, particularly in the United States, considers these minimum essentials of the mathematics of investment an indispensable part of the equipment of any person who wishes to deal intelligently with the modern business world.

There is little of novelty in the 30 pages of theory with which the author starts the book. He treats simple interest, simple discount, compound interest, and annuities certain in orthodox fashion. The most general annuity problem which he considers is one where the interest period involved is an integral multiple of the payment interval of the annuity. The author does not introduce any of the customary actuarial abbreviations $a_{\overline{n}|}$, $s_{\overline{n}|}$, etc., but leaves his formulas in their inconvenient explicit algebraic forms. Apparently, he does not adopt the desirable logical attitude of considering simple discount, or simple interest payable in advance, as an entity by itself. He considers it as an *approximation to simple interest* which is convenient in certain applications but which, nevertheless, is slightly unjust to the borrower.

After the brief consideration of theory, the remainder of the text (about 30 pages) is devoted to applications. There is one chapter dealing with the amortization of debts, with remarks included concerning the sinking fund method—which, it is interesting to note, is referred to as the "système américain." A second chapter is devoted to a discussion of loans (including bonds as a

special case) considered from the standpoint of an investor who buys the title to some of the debtor's payments. The main object of the chapter is to introduce an interpolation method for determining the yield which such an investor obtains from his purchase. In the case of a bond which is a part of an issue which is redeemable in installments or by drawings, the author makes extensive use of a notion which he refers to as the *mean* price of a bond of the issue, at a given yield. Also, he introduces the companion notion of the *mean* yield, in case a bond of such an issue is bought at a specified price.

In an appendix there are twelve pages of brief extracts of the usual tables employed in the mathematics of investment. In this appendix there is also given a brief discussion of the theory of exponents and of geometric progressions. Logarithms are not explicitly used in the book. Throughout the book the author gives a sufficient number of illustrative examples, solved in the text, to clarify the general processes introduced. However, there are no exercises included for the reader to solve. The mathematical manipulations in the text are performed accurately, but the methods employed present no novelty.

The reviewer believes that the book as a whole would be interesting to an American reader not because of any particular novelty in its content, but because it furnishes an illustration of the courses in the mathematics of investment which are presented to the general student in French schools of business administration.

WILLIAM L. HART

The Logic of Discovery. By R. D. Carmichael. The Open Court Publishing Company, Chicago, 1930. ix+280 pages. Price $2.00.

The scope of this book is somewhat broader than might be inferred from the title. The volume, parts of which have been previously published in various periodicals, gives a careful analysis of some phases of the philosophy of mathematics, with especial reference to their possible influence on the philosophy of other sciences. As stated in the preface, no technical knowledge of mathematics on the part of the reader is presupposed.

The first chapter, "The Logic of Discovery," discusses the possibility of developing a systematic analysis of those mental processes by which an investigator is led to infer the probable truth of a proposition before he is able to give a demonstrative proof of its validity.

In this chapter the reader's attention is directed to a theory which is certainly very interesting, and which, if firmly established, must have profound implications in the philosophy of science. In discussing "the locus of the essential step in discovery," the author says that "It is in the formation of the conjecture itself or goes back even farther to the formation of the hypothesis out of which comes the proposition to be tested, whether by experiment or by reasoning. It may even be found in a more remote place in the process of discovery than this, its chief element resting in a principle partaking somewhat of a metaphysical nature (as in the general principle of relativity) or in an ideal of a purely abstract character (as in Descartes' doctrine of clarity). It is as if the

mind were seeking to impose itself upon nature, insisting that whatever explanations we may finally adopt, they shall be such as satisfy the requirements of a norm set up by the mind itself. Certain of these demands may be impossible of realization. One then constructs a norm of a modified sort. The essential step in discovery is in the construction of definite hypothesis in the form of a particular or a general law or proposition and in the formulation of a principle or norm lying back of the hypothesis and contributing effectively to giving it existence." (Pages 13, 14.) This conception of scientific truth as depending upon the mental processes of the observer as well as upon the objective facts of nature is one which is repeatedly emphasized throughout the book.

In Chapter II, "What is the Place of Postulate Systems in the Further Progress of Thought?," the author explains briefly the use of postulate systems, and shows that although this name is most commonly used in connection with purely mathematical theories, essentially the same principle is involved in the development of certain branches of other sciences, notably in rational mechanics.

Chapter III, "On the Nature of Systems of Postulates," probably requires of the non-mathematical reader a greater degree of intellectual effort than any other in the book. The author has, however, succeeded admirably in minimizing the difficulties of the subject. A simple postulate system is studied in detail. The exposition is carefully planned to yield a large amount of information with a minimum of technique.

Undoubtedly some authorities will consider that Professor Carmichael's explanation of what is meant by "rigorous thinking" (on pages 76 and 77) places too much emphasis on the postulational formulation of the foundations and not enough on the necessity for careful discrimination in the choice of methods by which one builds on this foundation. Just what mental processes imply logical necessity is a question on which there is by no means unanimous agreement among scholars. The author's discussion does, however, serve to show the necessity for explicit statement of all hypotheses in a logical exposition of any scientific theory.

In Chapter IV, "Concerning the Postulational Treatment of Empirical Truth," the author discusses in greater detail than previously the possibility of a more explicit use of the postulational method in the development of scientific theories. Economics is given particular attention, and an ingenious plan (due to Rueff) for the reduction of this science to a purely deductive form is described. The possibility of such a treatment of ethics is also considered.

In discussing the procedure to be followed if it is found that a particular system of postulates leads to results which are invalid for a set of objects under investigation, the author says: "We shall need to replace our given system by one which is not equivalent to it; moreover, we shall have to replace it by one having no concrete interpretation (of any sort) in common with the one which we seek to modify so as to avoid contradiction—a fact which will be apparent on a little reflection." (Pages 89, 90.) It may serve to make the situation somewhat clearer if it is noted that in the part of the sentence following the word "moreover" the author evidently has in mind the case in which the new system

(but not necessarily the original one) is categorical. Without this qualification, the statement is incorrect, as may be shown by a simple example.[1] On the whole, however, the exposition is logical and clear.

In Chapter V, "The Structure of Exact Thought," and in Chapter VI, "The Notion of Doctrinal Function," the author continues the discussion of postulate theory. Here special emphasis is given to the fact that the development of a mathematical science associated with a system of postulates is entirely independent of any particular concrete interpretation of the system. The theory is illustrated by application to two specific postulate systems, one of which states the fundamental properties of finite groups.

Chapter VII, "Hypothesis Growing into Veritable Principle," contains an account of the evolution of scientific ideas from vague speculative conjectures, based on little or no experimental evidence, to veritable principles of science. An excellent example of this process is furnished by the atomic theory, the history of which the author traces from the philosophical speculations of the Greeks to the electronic theory of matter.

The chapter also contains an able criticism of Delacre's view of value of abstract theory in scientific research, and a discussion of Vaihinger's philosophy of the "As If."

Chapter VIII, "What is Reasoning?," will perhaps be the most interesting in the book to a reader with an interest in psychology. This chapter is closely connected with the first. It gives an interesting discussion of the rôle of the subconscious mind in mathematical discovery.

The author shows in a convincing manner that the theory advanced in Rignano's *Psychology of Reasoning* is entirely inadequate to account for such logical processes as mathematical induction, or the type of thought involved in the construction of a doctrinal function.

The closing chapter, "The Larger Human Worth of Mathematics," is devoted to a discussion of some aspects of the cultural value of mathematics.

The book is written in a very readable style and is heartily recommended to anyone who is anxious to become familiar with the interrelations of mathematics, the physical and social sciences, and philosophy.

FRED W. PERKINS

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[1] A system S comprising the postulates numbered I–VI, inclusive, (on pages 62 and 63) and VII_2 (page 72) with $n=2$, is consistent. A second system S', obtained from S by replacing VII_2 by VII_1 (page 72) with $n=2$, is consistent and non-categorical. If the set of objects to be studied are the seven letters $A, B, C \cdots G$, arranged in m-classes as indicated on page 64, then the system S is inapplicable, but the system S' is applicable. Nevertheless S and S' have a concrete interpretation in common, namely the thirteen letters $A, B, C, \cdots, M$ arranged in m-classes as indicated on page 71.

That the statement given in the text is correct when qualified as noted in this review, is readily established by noting that if the original system and a new categorical system have one concrete interpretation in common, then (since all interpretations of the new system are isomorphic) the original system must be applicable to all concrete interpretations of the new system; in other words, the new system cannot be valid for any set of objects for which the original system is invalid.

PROBLEMS AND SOLUTIONS

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. L. OLSON

Send all communications about Problems and Solutions to B. F. Finkel, Springfield, Mo. All manuscript should be typewritten, with double spacing, and with margins at least one inch wide

PROBLEMS FOR SOLUTION

N. B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known textbooks or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association in the solution of such problems.

3509. *Proposed by R. Goormaghtigh, Bruges, Belgium.*

Through the vertices A, B, C of a triangle parallel lines are drawn cutting a given straight line Δ at α, β, γ; the parallels to BC, CA, AB drawn through α, β, γ form a triangle $A_1B_1C_1$ equal to ABC, (Monthly, 1929, p. 424). Prove that the isogonal conjugates to B_1C_1, C_1A_1, A_1B_1, in the angles $(\Delta, A\alpha)$, $(\Delta, B\beta)$, $(\Delta, C\gamma)$ respectively are concurrent.

3510. *Proposed by William Sell, University of Alabama.*

Ellipses, as many as possible, are drawn wholly within a circular plane region R. Each ellipse has semi-axes a and b. No two ellipses overlap, and each is tangent to at least two others, but otherwise they are placed at random. The probability of the inclination (of any major axis to a fixed line) having a particular value is the same for all values from zero to 2π. Find the limit of the quotient of the area of R divided by the sum of the areas of the ellipses, as R increases without limit.

3511. *Proposed by W. H. Rasche, Virginia Polytechnic Institute.*

Given an irregular tetrahedron and a triangle; show how to pass a plane which shall cut from the tetrahedron a triangle similar to the given triangle.

3512. *Proposed by J. Rosenbaum, Milford, Conn.*

Prove that in the tetrahedron of Problem 3482 (March, 1931), the common center of the two spheres is also the centroid of the tetrahedron.

3513. *Proposed by L. S. Johnston, University of Detroit.*

Given

$$f(s, p) = \sum_{k=1}^{p} (-1)^{p+k}(2k-1)_sH_{p-k}$$

where

$$_sH_r \equiv \frac{s(s+1)(s+2)\cdots(s+r-1)}{r!}, \quad _sH_0 \equiv 1.$$

Prove: (a) If p is odd, then $f(s, p)$ is positive for all values of s;

(b) If p is even, then $f(s, p)$ is positive, equal to zero, or negative according as s is less than, equal to, or greater than 3.

3514. *Proposed by J. P. Ballantine, University of Washington.*

Let D_n denote a determinant of order n whose elements are all zeros and ones, and which has 2 ones and $n-2$ zeros in every row and column. Show that:

(a) For every n, $D_n = \pm 2^m$, where m and n are both even or both odd, or $D_n = 0$.

(b) If $D_5 = 0$, then two rows are identical, and conversely.

(c) If two rows of D_n are alike, then two columns are alike, and conversely.

(d) $3m \leqq n$,

(e) Show that property (*b*) does not hold except for D_2, D_3, and D_5.

3515. *Proposed by E. P. Bogdanoff, Harbin City, China.*

Give an elementary proof, without the use of the calculus, that the equation

$$2^x = 4 \cdot x$$

has two real roots only, and calculate each of them.

Determine the location of the imaginary roots, and compute the pair having the least absolute value.

SOLUTIONS

299 [1914, 267; 1931, 171]. *Proposed by B. F. Finkel, Drury College.*

A cone rests in two fluids which do not mix, with its vertex downwards and its base in the surface of the upper fluid; to find how much its density must be increased, that it may rest with its base in the common surface of the fluids.

From Walton's *Hydrostatical Problems.*

Solution by Harry D. Ruderman, Brooklyn, N. Y., and the Proposer

Let σ_1 be the density of the upper fluid; σ_2, the density of the lower fluid; ρ_1, the density of the cone in the first position; and ρ_2, the density of the cone in the second position.

Since the cone in the second position is to have the surface of its base coincident with surface of the lower fluid, the density of the cone in the second position must be the same as the density of the lower fluid, that is, $\rho_2 = \sigma_2$, and hence, if the altitude of the cone does not exceed the depth of the upper fluid, $\rho_2 - \rho_1 = \sigma_2 - \sigma_1$ is the amount the density of the cone must be increased to satisfy the conditions of the problem.

In the contrary case let h_1 be the length of the whole altitude of the cone and h_2 the length of that portion of the altitude immersed in the second fluid when the cone is in its first position. Let r_1 be the radius of the base of the cone and r_2, the radius of the section of the cone formed by the common surface of the fluids.

Now the mass of the cone must be equal to the masses of the two fluids displaced by the cone.

The mass of the cone satisfying the first condition of the problem is $\frac{1}{3}\pi r_1^2 h_1 \rho_1$. The mass of the first liquid displaced by the cone is

$$\tfrac{1}{3}\pi(r_1^2 + r_1r_2 + r_2^2)(h_1 - h_2)\sigma_1$$

and the mass of the second liquid displaced is $\tfrac{1}{3}\pi r_2^2 h_2\sigma_2$.

Hence

$$\tfrac{1}{3}\pi r_1^2 h_1\rho_1 = \tfrac{1}{3}\pi(r_1^2 + r_1r_2 + r_2^2)(h_1 - h_2)\sigma_1 + \tfrac{1}{3}\pi r_2^2 h_2\sigma_2;$$

whence $r_1^2 h_1\rho_1 = (r_1^2 + r_1r_2 + r_2^2)(h_1 - h_2)\sigma_1 + r_2^2 h_2\sigma_2$. From similar triangles, $r_2/r_1 = h_2/h_1$ or $r_2 = r_1h_2/h_1$ substituting r_1h_2/h_1 for r_2 in the above equation and solving for ρ_1 we have

$$\rho_1 = \sigma_1 \frac{[h_1^2 + h_1h_2 + h_2^2]}{h_1^3}(h_1 - h_2) + \frac{h_2^3}{h_1^3}\sigma_2$$

or

$$\rho_1 = \sigma_1(h_1^3 - h_2^3)/h_1^3 + \frac{h_2^3}{h_1^3}\sigma_2$$

From the equation $\rho_2 = \sigma_2$ above, we subtract the equation for ρ_1 and we have

$$\rho_2 - \rho_1 = \sigma_2 - \frac{h_2^3}{h_1^3}\sigma_2 - \frac{h_1^3 - h_2^3}{h_1^3}\sigma_1$$

or

$$\rho_2 - \rho_1 = (\sigma_2 - \sigma_1)(h_1^3 - h_2^3)/h_1^3.$$

Also solved by B. D. Roberts.

3239 [1927, 98]. *Proposed by Alex S. Wiener, Cornell University.*

Solve the following simultaneous equations for u, v, w, x, y, and z:

$$(u - a_1)(x - b_1) - (v - b_1)(w - a_1) = 0,$$
$$(u - a_2)(z - b_2) - (v - b_2)(y - a_2) = 0,$$
$$(w - a_3)(z - b_3) - (x - b_3)(y - a_3) = 0,$$
$$u^2 + v^2 = w^2 + x^2 = y^2 + z^2 = r^2.$$

II. Solution by Otto Dunkel, Washington University.

This second solution is given since the first one [1929, 288] is deficient in some respects and since it involves unnecessarily tedious algebraic computations. This solution applies to the more general problem in which the circle is replaced by any conic, and it does not require a change of coordinates. It may be of interest since it proves incidentally some known geometric theorems. We first restate the generalized problem in geometric form. Given any conic S and three points in its plane $A_1(a_1, b_1)$; $A_2(a_2, b_2)$; $A_3(a_3, b_3)$; to find the coordinates (y, z); (w, x); (u, v); of the vertices C_1, C_2, C_3 of a triangle inscribed in S such that C_1C_2 passes through A_3; C_2C_3, through A_1; C_3C_1, through A_2. We shall discard the trivial case of a vertex of $A_1A_2A_3$ lying upon S.

An exceptional case which was not mentioned in the first solution will be considered first. If $A_1A_2A_3$ is a self-polar triangle with respect to S there is an infinite number of solutions obtained by taking any point of S for one vertex of the required triangle. For, if A_3 is the vertex inside the conic, let C_1 be a point on S, and draw C_1A_3 cutting S again in C_2. Draw A_1C_2 cutting S again in C_3. Since A_1A_2 is the polar of A_3, the pencil $A_2(C_2\,A_3\,C_1\,A_1)$ is harmonic; since A_2A_3 is the polar of A_1, the pencil $A_2(C_2\,A_3\,C_3\,A_1)$ is harmonic. Hence A_2C_1 and A_2C_3 lie on the same straight line, or A_2C_1 and A_1C_2 meet in C_3 on S. Similarly A_2C_2 and A_1C_1 meet in C_3' on S, and C_3C_3' passes through A_3. Hence any one of the four triangles with vertices at C_1, C_2, C_3, C_3' is a required triangle.

We take up now the case where A_1, A_2, A_3 do not lie in a straight line and do not form a self-polar triangle. Let $A_1'A_2'A_3'$ be the triangle such that $A_2'A_3'$ is the polar of A_1; $A_3'A_1'$, of A_2; $A_1'A_2'$, of A_3. Denote by $P(xx)=0$ the equation of the conic S, and by $P(ix)=0$ the equation of the polar of $A_i(a_i, b_i)$. These two equations contain both x and y but for brevity of writing we drop the y in the notation. We have $P(ij)=P(ji)$ and $P(ii)\neq 0$, $i=1, 2, 3$. Obviously the equations of the lines joining corresponding vertices are

$$
\begin{aligned}
A_1A_1' &: P(31)P(2x) - P(12)P(3x) = 0,\\
A_2A_2' &: P(12)P(3x) - P(23)P(1x) = 0,\\
A_3A_3' &: P(23)P(1x) - P(31)P(2x) = 0.
\end{aligned}
\tag{1}
$$

These equations give immediately the theorem that the triangles $A_1A_2A_3$ and $A_1'A_2'A_3'$ are perspective.

Let A_1A_1' cut $A_2'A_3'$ in A_1'', and let A_2'', A_3'' be defined similarly. The equation of $A_2''A_3''$ will now be derived. This equation may be written in two ways

$$
\begin{aligned}
&- P(12)P(3x) + P(23)P(1x) - kP(2x) = 0,\\
&P(23)P(1x) - P(31)P(2x) - lP(3x) = 0.
\end{aligned}
$$

It is obvious that each of these equations is the equation of $A_2''A_3''$ if $k=P(31)$ and $l=P(12)$. Hence we have

$$
\begin{aligned}
A_2''A_3'' &: P(12)P(3x) - P(23)P(1x) + P(31)P(2x) = 0,\\
A_3''A_1'' &: P(23)P(1x) - P(31)P(2x) + P(12)P(3x) = 0,\\
A_1''A_2'' &: P(31)P(2x) - P(12)P(3x) + P(23)P(1x) = 0.
\end{aligned}
\tag{2}
$$

These equations tell us that the pencils of lines such as $A_2''A_3''$, $A_3''A_1''$, $A_1'A_2'$, A_3A_3' are harmonic. The two lines $A_2''A_3''$, $A_3''A_1''$ and the conic determine a set of conics having four points in common. The equations of such conics may be written

$$
P^2(12)P^2(3x) - [P(23)P(1x) - P(31)P(2x)]^2 - kP(xx) = 0.
$$

We find the conic of the set which passes through A_3 by setting $x=a_3$, $y=b_3$,

and we thus find $k = P^2(12)P(33)$, since $P(33) \neq 0$. Hence the equation of this conic is

$$(3) \qquad P^2(12)[P^2(3x) - P(33)P(xx)] - [P(23)P(1x) - P(31)P(2x)]^2 = 0.$$

Since $P(3x) = 0$ is the polar of A_3 with respect to $P(xx) = 0$, the first bracket is the product of two linear factors α and β. Hence (3) has the form $P^2(12)\alpha\beta - \gamma^2 = 0$, where α, β, γ each vanish for a_3, b_3. Therefore (3) is the equation of two straight lines intersecting in A_3 and passing through pairs of points in which $A_2'' A_3''$, $A_3'' A_1''$ cut S. Denote these two lines by $C_1 A_3 C_2$ and $C_1' A_3 C_2'$, where C_1, C_1' are the intersections of $A_2'' A_3''$ with S, and C_2, C_2', those of $A_3'' A_1''$ with S. In a similar manner we define C_3 and C_3' on $A_1'' A_2''$, and we then have the following sets of three points on a straight line

$$(4) \qquad \begin{aligned} &C_2 A_1 C_3, \ C_3 A_2 C_1, \ C_1 A_3 C_2, \\ &C_2' A_1 C_3', \ C_3' A_2 C_1', \ C_1' A_3 C_2', \end{aligned}$$

where all the C's lie on S.

The equations (2) taken in turn with $P(xx) = 0$ determine the coordinates of the points C_1, C_2, C_3 independently of the manner in which they were obtained. If $A_1A_2A_3$ is self-polar $P(12) = P(23) = P(31) = 0$, and we have no equations. But this is the case of an infinite number of solutions which we have already considered and which is easily handled. If the three points A_1, A_2, A_3 lie in a straight line, $P(1x)$, $P(2x)$ and $P(3x)$ are linearly dependent, and we must examine the equations (2) in this regard. Consider the first one and suppose that it is identically zero; then by setting the x and y in it equal to the coordinates of the vertices of the triangle $A_1A_2A_3$ we find in turn $P(23)P(11) = 2P(12)P(31)$, $P(31)P(22) = 0$, $P(12)P(33) = 0$. Or $P(23) = P(31) = P(12) = 0$, and this means that $A_1A_2A_3$ is self polar. Hence no equation of (2) is identically zero, if the triangle is not self-polar; and these equations determine the solution in all cases except the case of an infinite number of solutions. We now see that there are an infinite number of solutions only when the given triangle is self-polar.

The equations (2) show that the straight line

$$P(12)P(3x) + P(23)P(1x) + P(31)P(2x) = 0$$

is the axis of perspectivity on which the corresponding sides of $A_1' A_2' A_3'$ and $A_1'' A_2'' A_3''$ meet. All of the above results may be obtained by synthetic projective geometry methods.

3391 [1929, 448]. *Proposed by Otto Dunkel, Washington University.*

The ratio of the shortest diagonal to a side of an ordinary regular polygon of 19 sides satisfies an equation of the 9th degree with rational coefficients. The remaining roots are the corresponding ratios with alternately plus and minus signs for the remaining regular polygons of star form. By a known theorem the roots of this equation may be obtained by solving first a cubic with rational

coefficients and then solving three other cubics whose coefficients are rational functions of the roots of the first cubic. Derive with as little computation as possible a set of such equations.

Solution by the Proposer.

In the case of the convex regular polygon of 19 sides the equation for the ratio of the shortest diagonal to a side may be written down from the solution of 3322 [1929, 291–293] by equating the lengths of the diagonals $A_1A_n = y_n$ for $n=10$ and $n=11$. Using the formula in the note on that solution for y_n, we obtain the equation

$$y^9 - y^8 - 8y^7 + 7y^6 + 21y^5 - 15y^4 - 20y^3 + 10y^2 + 5y - 1 = 0, \tag{1}$$

with the root $y_3/y_2 = y$. This equation results from the use of the formula in the note

$$y_n' = yy_{n-1}' - y_{n-2}', \tag{2}$$

where $y_n' = y_n/y_2$. The star formed polygon with the side y_j, $j=3, 4, \cdots, 10$, has y_{2j-1} for its corresponding diagonal, and it is to be shown that $(-1)^j y_{2j-1}/y_j$ is also a root of (1). Let $A_1', A_2', \cdots, A_{19}'$ denote the successive vertices of the polygon, where $A_1'A_2' = A_1A_j$, $A_1'A_3' = A_1A_{2j-1}$, $A_1'A_n' = A_1A_{(n-1)j-(n-2)}$. If r is the radius of the circle circumscribing the polygon and $\theta = \pi/19$, $A_1'A_n' = 2r \sin (n-1)(j-1)\theta$. We have

$$\sin (n-1)(j-1)\theta + \sin (n-3)(j-1)\theta = 2 \sin (n-2)(j-1)\theta \cos (j-1)\theta,$$

$$= \frac{\sin (n-2)(j-1)\theta \sin 2(j-1)\theta}{\sin (j-1)\theta}$$

or

$$\frac{A_1'A_n'}{A_1'A_2'} + \frac{A_1'A_{n-2}'}{A_1'A_2'} = \frac{A_1'A_{n-1}'}{A_1'A_2'} \frac{A_1'A_3'}{A_1'A_2'},$$

$A_1'A_2' = 2r \sin (j-1)\theta$, $A_1'A_3' = 2r \sin 2(j-1)\theta$, $0 < 2(j-1)\theta \leqq 18\pi/19$. $A_1'A_2'$ and $A_1'A_3'$ are both positive, but $A_1'A_n'$ may be negative for certain values of n. Hence the equation (2) is satisfied when $A_1'A_n'/A_1'A_2'$ is substituted for y_n' and $A_1'A_3'/A_1'A_2'$ for y. But since $10(j-1)\theta + 9(j-1)\theta = (j-1)\pi$, we have $A_1'A_{11}' = (-1)^j A_1'A_{10}'$. Since y_{11}' contains only odd powers of y and y_{10}' only even powers, a solution of (1) is $(-1)^j A_1'A_3'/A_1'A_2' = (-1)^j A_1A_{2j-1}/A_1A_j = (-1)^j 2 \cos (j-1)\theta$.

It will be more convenient to consider the equation

$$y^9 + y^8 - 8y^7 - 7y^6 + 21y^5 + 15y^4 - 20y^3 - 10y^2 + 5y + 1 = 0, \tag{3}$$

whose roots are the negatives of the roots of equation (1). Hence the roots of (3) are

$$
\begin{aligned}
&r_1 = 2\cos 2\theta, \quad r_8 = -2\cos 3\theta, \quad r_7 = -2\cos 5\theta,\\
&r_2 = 2\cos 4\theta, \quad r_3 = 2\cos 6\theta, \quad r_5 = -2\cos 9\theta,\\
&r_4 = 2\cos 8\theta, \quad r_6 = -2\cos 7\theta, \quad r_9 = -2\cos\theta,\\
&r_k = \epsilon^k + \epsilon^{-k}, \quad \epsilon = e^{2\theta i}, \qquad \theta = \pi/19.
\end{aligned} \tag{4}
$$

The order of writing these roots is determined as follows: the numbers in the first line below are congruent, modulus 19, to the corresponding numbers in the second line

$$
\begin{array}{cccccccccc}
1, & 2, & 4, & 8, & -3, & -6, & 7, & -5, & 9,\\
2^0, & 2^1, & 2^2, & 2^3, & 2^4, & 2^5, & 2^6, & 2^7, & 2^8.
\end{array} \tag{5}
$$

Now set

$$z_1 = r_1 + r_8 + r_7, \quad z_2 = r_2 + r_3 + r_5, \quad z_3 = r_4 + r_6 + r_9; \tag{6}$$

and let us find the equation whose roots are the z's. We have at once

$$z_1 + z_2 + z_3 = -1, \tag{7}$$

since the left side is the sum of the roots of (3). We next calculate the products z_1z_2, z_2z_3, z_1z_3. This may be conveniently done as follows for z_1z_2. From (4) it will be seen that the subscripts of the r's are the exponents of ϵ, and we have merely to write the sum and the difference of every pair of subscripts taking one from z_1 and its mate from z_2. Thus we get

3, 10, 9, 4, 11, 10, 6, 13, 12, 1, 6, 5, 2, 5, 4, 4, 3, 2.

The numbers 10, 11, 12, 13 are then replaced by the absolute values of their residues with respect to 19, i.e., 9, 8, 7. 6. We then count the number of times the sets of subscripts, 1, 8, 7; 2, 3, 5; 4, 6, 9 occur. Hence

$$z_1z_2 = z_1 + 2z_2 + 3z_3; \quad z_2z_3 = z_2 + 2z_3 + 3z_1; \quad z_3z_1 = z_3 + 2z_1 + 3z_2. \tag{8}$$

Then from (7) we obtain $\Sigma z_iz_j = -6$. In order to evaluate $z_1z_2z_3$ we eliminate z_3 from the expression for z_1z_2 by means of (7), and thus find $z_1z_2 = -z_2 - 2z_1 - 3$. Multiplying by z_3 and then eliminating the two products by means of (8) we obtain

$$z_1z_2z_3 = -7(z_1 + z_2 + z_3) = 7. \tag{9}$$

Hence the z's satisfy the equation

$$z^3 + z^2 - 6z - 7 = 0. \tag{10}$$

All the roots of this equation are real, and it is obvious that it has one and only one positive root. In order to identify the roots we observe that $\frac{1}{2}z_2$ contains two positive terms $\cos 4\theta$, $\cos 6\theta$, and the negative term $-\cos 9\theta$, and that $\cos 4\theta > \cos 9\theta$. Hence the positive root is z_2, and z_1 and z_3 are each negative. From $\frac{1}{2}(z_1 - z_3) = (\cos\theta - \cos 3\theta) + (\cos 2\theta - \cos 5\theta) + (\cos 7\theta - \cos 8\theta)$, we see that each parenthesis on the right is positive. Hence $z_1 > z_3$.

We shall now obtain the three equations for the three sets of r's. We have $r_1+r_7+r_8=z_1$. Also $r_1r_7=r_8+r_6$; $r_1r_8=r_9+r_7$; $r_7r_8=r_{15}+r_1=r_4+r_1$. We add and subtract the two subscripts in the product to get the two terms on the right. Hence $\Sigma r_ir_j=r_1+r_8+r_7+r_4+r_6+r_9=z_1+z_3$. Then $r_1r_7r_8=r_8^2+r_6r_8=r_{16}+r_{14}+r_0+r_2=2+r_2+r_3+r_5=2+z_2$. Hence the equation for r_1, r_7, r_8 is

$$y^3 - z_1y^2 - (1 + z_2)y - (2 + z_2) = 0, \quad r_1, r_7, r_8. \tag{11}$$

The other two equations are obtained in the same manner

$$\begin{aligned} &y^3 - z_2y^2 - (1 + z_3)y - (2 + z_3) = 0, \quad r_2, r_3, r_5, \\ &y^3 - z_3y^2 - (1 + z_1)y - (2 + z_1) = 0, \quad r_4, r_6, r_9. \end{aligned} \tag{11'}$$

There are other simple ways of deriving equation (10). From the equation used to establish (9) and from two similar equations we have

$$z_2 = -2 - \frac{1}{z_1 + 1}; \quad z_3 = -2 - \frac{1}{z_2 + 1}; \quad z_1 = -2 - \frac{1}{z_3 + 1}. \tag{12}$$

It may also be shown, without the use of (10), that

$$z_2 = 4 - z_1^2; \quad z_3 = 4 - z_2^2; \quad z_1 = 4 - z_3^2. \tag{13}$$

It suffices then to find one root of (10), say z_1, and then z_2 may be calculated from the first equation of either (12) or (13).

It will be easily seen that the equation

$$y_n' = yy_{n-1}' - y_{n-2}'$$

gives a geometrical interpretation of the usual process for reducing a reciprocal equation of even degree to an equation of half its degree.

3431 [1930, 261]. *Proposed by W. H. Rasche, Virginia Polytechnic Institute.*

The letters a and b designate respectively two fixed, non-complanar lines, the angle between which is α, and the perpendicular distance between which is l; the letters P and Q designate two points on a third line c such that the distance PQ is constant; moreover, points P and Q of line c move on lines a and b, respectively. Show: (I) that the locus (relative to the frame formed by lines a and b) of any third point S of line c is an ellipse whose plane is parallel to lines a and b and whose semi-major and semi-minor axes are

$$\left\{\frac{(n-m)^2 - l^2}{4\sin^2\alpha} + nm\frac{(n-m)^2 - l^2}{(n-m)^2}\right\}^{1/2} + \frac{\{(n-m)^2 - l^2\}^{1/2}}{2\sin\alpha} \tag{1}$$

and

$$\left\{\frac{(n-m)^2 - l^2}{4\sin^2\alpha} + nm\frac{(n-m)^2 - l^2}{(n-m)^2}\right\}^{1/2} - \frac{\{(n-m)^2 - l^2\}^{1/2}}{2\sin\alpha}, \tag{2}$$

wherein n and m represent the linear segments QS and PS, respectively; (II) that the angles which lines a and b make with the semi-major and the semi-minor axes, respectively, are

$$\gamma = 45° - \tfrac{1}{2}(\alpha + \beta), \tag{3}$$

and

$$\delta = 45° - \tfrac{1}{2}(\alpha - \beta), \tag{4}$$

wherein β is given by the relation

$$\tan \beta = (n - m) \cot \alpha/(n + m). \tag{5}$$

Note: The linear segments n and l of expressions (1) and (2) are regarded as positive for all configurations; m in these expressions is positive or negative according as point S divides segment PQ externally or internally; angles α and β, in equations (3), (4), and (5), are regarded as positive for all configurations; γ and δ may be either positive or negative, depending upon the position of point S on line c.

Solution by Otto Dunkel, Washington University.

Let a and b cut their common perpendicular in A and B, where $AB = l$; and let the plane through S perpendicular to AB cut the latter in O. Then $OA/OB = m/n$, and S moves in this fixed plane. Let the projections on this plane of P, Q, a, b, c be P', Q', a', b', c', and set $P'S = p$, $Q'S = q$. Then P' and Q' lie on the fixed lines a' and b', respectively, and we have

$$p/q = m/n, \qquad q - p = [(n - m)^2 - l^2]^{1/2}$$
$$p = \frac{m}{n - m}(q - p), \quad q = \frac{n}{n - m}(q - p). \tag{1}$$

Thus the problem reduces to finding the locus of S, where S, P', Q' are fixed points on the moving line c' such that P' and Q' move on the fixed straight lines a' and b', respectively. It will now be shown how to reduce this construction for S to the ordinary trammel construction for an ellipse. Let a circle (C) be passed through P', Q', O for a given position of c', and suppose that this circle is now rigidly attached to c' at P' and Q' and moves with it. Since the fixed chord $P'Q'$ of (C) subtends the angle α, which is the angle between a' and b', at every point of a certain arc of (C) and since P' and Q' lie on a' and b', the point O must lie on (C) for every position of c'. Through S and C, the center of (C), draw a straight line cutting (C) in X and Y. Draw also the lines $x = OX$, $y = OY$ on the fixed plane, and suppose that XYS is also rigidly attached to the moving system of c' and (C). The arc XP' subtends a constant angle at points on part of the circumference of (C) and this angle is the same in the initial position as the properly chosen angle between a' and x, intersecting in the point O on (C). Since P' lies always on a', X must lie always on x, since it does so for the initial posi-

tion. Similarly Y moves on y. We now have a straight line carrying fixed points on it X, Y, S so that X moves on x, Y on y, and x and y meet in a right angle at O. This is the trammel construction for the ellipse. Consider now the simple case where n and m are both positive and $n>m$; then p and q are positive and $q>p$. Suppose also that X and Y have been so chosen that YS has a greater length than XS. Then the semi-major axis has the length of YS on the x axis, and the semi-minor axis has the length of XS on the y axis. Let r be the radius of (C), M the mid-point of $P'Q'$, β the acute angle CSM, γ the angle between x and a', and δ the angle between y and b'. Then $\angle MCP'=\alpha$, $\angle XCP'=2\gamma$ and $\angle YCQ'=2\delta$, and we easily find

$$2r\sin\alpha = q - p, \qquad \tan\beta = \frac{q-p}{q+p}\cot\alpha, \tag{2}$$
$$\gamma = 45^\circ - \tfrac{1}{2}(\alpha+\beta), \quad \delta = 45^\circ - \tfrac{1}{2}(\alpha-\beta).$$

In this case, if α is less than 90°, β may be considered as an acute positive angle, and then δ and γ are positive and acute. If, however, $\alpha>90^\circ$, we would naturally take β acute and negative and then γ and δ would be both negative. The lengths of the semi-axes are easily found from the equations

$$YS - XS = 2r = (q-p)/\sin\alpha, \quad YS\cdot XS = qp \tag{3}$$
$$YS + XS = \left[\left(\frac{q-p}{\sin\alpha}\right)^2 + 4pq\right]^{1/2}.$$

These results (1), (2), (3) give YS, XS, and $\tan\beta$ in terms of m and n which agree with the values in the problem. There is no difficulty in tracing the changes as S is taken at other points on c' relative to the segment $P'Q'$. When S lies within the segment $Q'P'$, q and p have opposite signs, as also m and n. Thus in this case the expression (2) of the problem becomes negative. If S lies between Y and C, or outside the segment on the Y side, the major axis lies upon y and not upon x as in the other case, so that γ now gives the angle between the minor axis and b.

3465 [1930, 551]. *Proposed by J. P. Dalton, University of Witwatersrand, Johannesburg, South Africa.*

Prove that for all positive integral values of n

$$\sum_{i=n}^{2n}(-1)^{2n-i}\frac{(2i+2)!}{i!(2n-i)!(2i-2n)!} = 2^{n-1}(n+1)(2n+1)(2n^2+7n+4).$$

I. *Solution by W. Randolph Church, Hightstown, New Jersey.*

We shall first prove that

$$\sum_{j=0}^{n}(-1)^j\binom{4n-2j+2}{2n+2}\binom{2n}{j} = 2^{2n-2}(2n^2+7n+4). \tag{1}$$

It is known (Netto, *Lehrbuch der Combinatorik*, §158) that

$$\sum_{s=0}^{q} \binom{p}{r+s}\binom{q}{s} = \binom{p+q}{r+q}.$$

Setting $p=4n-2j$, $r=2n$, $q=2$ in this formula, we have

$$\sum_{s=0}^{2} \binom{4n-2j}{2n+s}\binom{2}{s} = \binom{4n-2j+2}{2n+2}.$$

Substituting this in the left side of (1) we have

$$\sum_{j=0}^{n}(-1)^j\binom{4n-2j+2}{2n+2}\binom{2n}{j} = \sum_{j=0}^{n}(-1)^j \sum_{s=0}^{2}\binom{2}{s}\binom{4n-2j}{2n+s}\binom{2n}{j}$$

$$= \sum_{s=0}^{2}\binom{2}{s}\sum_{j=0}^{n}(-1)^j\binom{4n-2j}{2n+s}\binom{2n}{j}.$$

But it is also known (ibid, §160) that

$$\sum_{k=0}^{n}(-1)^k\binom{p}{k}\binom{2p-2k}{2p-m} = 2^m\binom{p}{m}.$$

Setting $p=2n$, $m=2n-s$ in this formula and substituting we have

$$\sum_{j=0}^{n}(-1)^j\binom{4n-2j+2}{2n+2}\binom{2n}{j} = \sum_{s=0}^{2}\binom{2}{s}2^{2n-s}\binom{2n}{2n-s}$$

$$= 2^{2n} + 2\cdot 2^{2n-1}\cdot 2n + 2^{2n-2}\cdot n(2n-1)$$

$$= 2^{2n-2}(2n^2+7n+4).$$

which completes the proof of (1).

Substituting $j=2n-i$ in (1), and changing the limits of summation accordingly, we obtain the formula

$$\sum_{i=n}^{2n}(-1)^{2n-i}\binom{2i+2}{2n+2}\binom{2n}{i} = 2^{2n-2}(2n^2+7n+4). \tag{2}$$

Writing (2) in terms of factorials, and introducing the factors $(2n+1)(2n+2)$ on each side, we have

$$\sum_{i=n}^{2n}(-1)^{2n-i}\frac{(2i+2)!(2n)!(2n+1)(2n+2)}{(2n+2)!(2i-2n)!i!(2n-i)!} = 2^{2n-1}(n+1)(2n+1)(2n^2+7n+4),$$

which reduces immediately to

$$\sum_{i=n}^{2n}(-1)^{2n-i}\frac{(2i+2)!}{i!(2n-i)!(2i-2n)!} = 2^{2n-1}(n+1)(2n+1)(2n^2+7n+4),$$

which is the given relation.

II. *Solution by Otto Dunkel, Washington University.*

This solution will employ the methods of the calculus of finite differences, and we first present some facts which will be used. The difference operator Δ may be written $\Delta = U-1$, $U^k f(x) = f(x+k)$. If we set $x^{(n)} = x(x-1)\cdots(x-n+1)$, then it is easily shown that

$$\Delta^p x^{(n)} = n(n-1)\cdots(n-p+1)x^{(n-p)}. \tag{1}$$

We now prove that

$$\Delta_x^n f(2x) = \left[\sum_{j=0}^{n} 2^{n-j}\binom{n}{j}\Delta_y^{n+j}\right] f(y), \quad y = 2x, \tag{2}$$

where the subscripts x and y indicate that the Δ applies only to the variable indicated by the subscript. Thus

$$\begin{aligned}\Delta_x f(2x) &= f(2x+2) - f(2x) = f(y+2) - f(y)\\ &= (U^2-1)f(y) = (U+1)(U-1)f(y) = 2\Delta(1+2^{-1}\Delta)f(y).\end{aligned}$$

Hence

$$\begin{aligned}\Delta_x^n f(2x) &= 2^n\Delta^n(1+2^{-1}\Delta)^n f(y),\\ &= \left[\sum_{j=0}^{n} 2^{n-j}\binom{n}{j}\Delta^{n+j}\right] f(y).\end{aligned}$$

We may write the sum on the left in the problem

$$\sum_{i=0}^{2n}(-1)^{2n-i}\binom{2n}{i}\phi(i), \quad (2n)!\phi(i) = (2i+2)^{(2n+2)}; \tag{3}$$

since $\phi(i) = 0$ if $i = 0, 1, \cdots n-1$. Now write (3) in the form

$$\left[\sum_{i=0}^{2n}(-1)^{2n-i}\binom{2n}{i}U^i\right]\phi(0) = (U-1)^{2n}\phi(0) = \Delta^{2n}\phi(0). \tag{4}$$

Since $\phi(i)$ is of degree $2n+2$ in i, $\Delta^{2n+j}\phi(i) = 0$ if $j > 2$. Hence by (2) we have

$$\Delta^{2n}\phi(0) = 2^{2n}\left[\Delta^{2n} + n\Delta^{2n+1} + \frac{n(2n-1)}{4}\Delta^{2n+2}\right]\phi(0),$$

where on the left the argument 0 is increased by two units in each operation of Δ, while on the right the increase is only one unit. Applying (1) to each of the three Δ operations on the right, we get after slight reductions

$$2^{2n}\frac{(2n+2)!}{(2n)!}\left[1 + 2n + \frac{n(2n-1)}{4}\right] = 2^{2n-1}(n+1)(2n+1)(2n^2+7n+4).$$

III. *Solution by the Proposer.*

The member on the left may be written

$$2(n+1)(2n+1)\sum_{j=0}^{n}(-1)^{n+j}\binom{2n+2j+2}{2j}\binom{2n}{n-j}.$$

In this the summed terms are the coefficient of x^{2n} in the expansion of $(1-x^2)^{2n}(1+x)^{-(2n+3)}$. This coefficient is equal to that of x^{2n} in the expansion of $(1-x)^{2n}(1+x)^{-3}$, that is to say,

$$\sum_{i=0}^{2n}\binom{i+2}{2}\binom{2n}{i}.$$

This readily reduces to $2^{2n-2}(2n^2+7n+4)$. Hence, the result given in the problem.

3470 [1931, 50]. *Proposed by F. L. Wren, George Peabody College for Teachers.*

If the hypotenuse of a right triangle be divided into n equal parts and the vertex of the right angle be joined to these points of equal division, then, if d_i be the length of the lines so drawn, we have

$$\sum_{i=1}^{n-1}d_i^2 = \frac{(n-1)(2n-1)}{6n}h^2,$$

where h is the length of the hypotenuse.

I. *Solution by Beatrice Aitchison, Johns Hopkins University.*

Take the x-axis along the side of the right triangle of length a and the y-axis along the side of length b. Then the coordinates of the ith point of division of the hypotenuse are $[(n-i)a/n, ib/n]$. Hence

$$\sum_{i=1}^{n-1}d_i^2 = \sum_{i=1}^{n-1}(n-i)^2a^2/n^2 + \sum_{i=1}^{n-1,}i^2b^2/n^2,$$

$$= \frac{a^2+b^2}{n^2}\sum_{i=1}^{n-1}i^2 = \frac{(n-1)(2n-1)}{6n}h^2.$$

II. *Generalization by R. Goormaghtigh, Bruges, Belgium.*

If the side BC of a triangle ABC, whose sides are a, b, c be divided into n equal parts and the vertex A be joined to these points, then if d_i be the length of the lines so drawn,

$$\sum_{i=1}^{n-1}d_i^2 = (n-1)bc\cos A + \tfrac{1}{6}n^{-1}(n-1)(2n-1)a^2.$$

For

$$d_i^2 = c^2(n-i)n^{-1} + b^2in^{-1} - a^2i(n-i)n^{-2}$$

and

$$\sum_{i=1}^{n-1} d_i^2 = c^2 n^{-1} \sum_{i=1}^{n-1} (n-i) + b^2 n^{-1} \sum_{i=1}^{n-1} i - a^2 n^{-1} \sum_{i=1}^{n-1} i + a^2 n^{-2} \sum_{i=1}^{n-1} i^2.$$

Hence

$$\sum_{i=1}^{n-1} d_i^2 = \tfrac{1}{2}(n-1)(b^2 + c^2 - a^2) + \tfrac{1}{6} n^{-1}(n-1)(2n-1)a^2$$

$$= (n-1)bc \cos A + \tfrac{1}{6} n^{-1}(n-1)(2n-1)a^2.$$

Also solved by Frank Ayres, R. P. Agnew, Jane Bent, E. M. Berry, Alice Bromwell, A. G. Clark, Mannis Charosh, Ralph Deutsch, Edward Fleischer, J. D. Hill, James Hamilton, L. S. Johnston, J. F. Locke, Clara Mize, G. T. Miller, J. H. Neelley, A. Pelletier, P. L. Rea, C. A. Rupp, O. J. Ramler, A. W. Randall, F. Underwood, F. B. Wiley, Paul Wernicke, Kamcheung Woo, G. A. Yanosik, B. F. Yanney and the Proposer.

3471 [1931, 50]. *Proposed by W. R. Ransom, Tufts College.*

In assigning dormitory rooms, a college gives preference to pairs of students in this order:

$$AA, AB, AC, BB, BC, AD, CC, BD, CD, DD,$$

in which AA means two seniors, AB means senior and junior, etc. Determine numerical values to assign to A, B, C, D so that the set of numbers $A+A$, $A+B$, $A+C$, $B+B$, etc., corresponding to the order indicated above, will be in descending magnitude. Find the general solution, and also the solution in least integers.

Solution by B. F. Yanney, College of Wooster.

The essential inequalities given, are as follows, the others being redundant:

$$(1) \qquad 2A > A + B > A + C > 2B, \quad B + C > A + D > 2C > B + D.$$

Obviously, inequalities 1, 2, and 4 of (1) imply that

$$A > B > C > D,$$

whence we may write:

$$(2) \qquad C = D + k_1, \quad B = D + k_1 + k_2, \quad A = D + k_1 + k_2 + k_3,$$

where the k's are all positive. These expressions substituted in inequalities 3, 4, 5, 6 of (1) give, respectively,

$$(3) \qquad k_3 > k_2, \quad k_1 > k_3, \quad k_2 + k_3 > k_1, \quad k_1 > k_2.$$

For the last inequality in (3), we may write $k_1 = k_2 + e$, which substituted in inequality 3 gives $k_3 > e$. For this last write $k_3 = e + e_1$. Then from inequality 2 we get $k_1 > e + e_1$, for which write $k_1 = e + e_1 + e_2 = k_2 + e$. Therefore $k_2 = e_1 + e_2$. This

in inequality 1 gives $k_3 > e_1 + e_2$, for which write $k_3 = e_1 + e_2 + e_3 = e + e_1$. Therefore $e = e_2 + e_3$. We have now secured the following values for the k's:

$$k_1 = e_1 + 2e_2 + e_3; \quad k_2 = e_1 + e_2; \quad k_3 = e_1 + e_2 + e_3. \tag{4}$$

Now, letting $D =$ an arbitrary number n, and substituting (4) in (2), we have the following general solution:

$$\begin{aligned} D &= n, \\ C &= n + e_1 + 2e_2 + e_3, \\ B &= n + 2e_1 + 3e_2 + e_3, \\ A &= n + 3e_1 + 4e_2 + 2e_3. \end{aligned} \tag{5}$$

We obtain the smallest integral values, if positive, by letting $n = e_1 = e_2 = e_3 = 1$; we have $D = 1$, $C = 5$, $B = 7$, $A = 10$.

Also solved by H. T. R. Aude, W. F. Cheney, A. G. Clark, S. A. Corey, R. F. Clash, Ralph Deutsch, Laurence Hampton, Theodore Lindqist, G. T. Miller, F. Underwood, G. A. Yanosik, F. L. Wilmer, and the Proposer.

3472 [1931, 51]. *Proposed by Morgan Ward, California Institute.*

Let S_n denote the sum of the nth powers of the roots of $F(x) = x^3 - Px^2 + Qx - 1$, where P, Q are integers; p, a prime of the form $3n+2$ chosen so that $F(x)$ is irreducible modulo p, and μ, the least value of n such that $S_n \equiv S_0$, $S_{n+1} \equiv S_1$, $S_{n+2} \equiv S_2$, mod p. Prove that if n is not divisible by μ, $S_n \equiv 0 \bmod p$, when and only when $S_{3n} \equiv 3 \bmod p$.

Solution by the Proposer.

In view of the identity $S_{3n} - 3 \equiv (S_n^2 - 3S_{-n})S_n$, it is sufficient to show that

$$S_n^2 - 3S_{-n} \equiv 0 \bmod p \tag{1}$$

only when μ divides n. If (1) is true, the polynomial $x^3 - S_n x^2 + S_{-n} x - 1$ whose roots are the nth powers of the roots of $F(x) = 0$ is congruent modulo p to

$$(x - \tfrac{1}{3}S_n)^3 + \frac{S_n^3}{27} - 1,$$

and hence is reducible modulo $p = 3n+2$, since, with respect to this modulus, every integer is a cubic residue.

The congruence $x^3 - S_n x^2 + S_{-n} x - 1 \equiv 0 \bmod p$ thus has a solution of the form

$$x = \alpha^n \equiv k \bmod p \tag{2}$$

where α is some root of $F(x) = 0$, and k is an integer.

Owing to the irreducibility of $F(x)$ modulo p, (2) implies that $\beta^n \equiv k$, $\gamma^n \equiv k \bmod p$, where β and γ are the remaining roots of $F(x) = 0$. Hence

$$\alpha^n \beta^n \gamma^n \equiv k^3 \bmod p, \quad \text{or} \quad k^3 \equiv 1, \; k \equiv 1 \bmod p.$$

But if $\alpha^n \equiv 1 \bmod p$, then $\alpha^{n+1} \equiv \alpha$, $\alpha^{n+2} \equiv \alpha^2$, $\alpha^{n+3} \equiv \alpha^3 \bmod p$ and $S_n \equiv S_0$, $S_{n+1} \equiv S_1$, $S_{n+2} \equiv S_2 \bmod p$. Accordingly, from the minimal character of μ, μ divides n.

3473 [1931, 51]. *Proposed by J. Rosenbaum, Milford, Connecticut.*

In the triangle ABC, the incircle is tangent to CA at D and to CB at E. Through a variable point P on DE, AP and BP are drawn meeting CB at X and CA at Y.

Find the envelope of the line XY.

Solution by E. M. Berry, Lynchburg College.

The following is a solution of the generalized problem: A triangle ABC has its sides tangent to a conic with D and E points of tangency on CA and CB respectively. Through a variable point P on DE, AP and BP are drawn meeting CB at X and CA at Y, find the envelope of the line XY.

We shall use homogeneous point and line coordinates. Take CDE as the triangle of reference. Let $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ be the equations of the sides EC, ED, and DC respectively then the vertices of the triangle CDE are $C(0:1:0)$, $D(1:0:0)$ and $E(0:0:1)$.

The general equation of the conic touching DC at D and EC at C is

$$2c_1x_1x_3 + c_2x_2^2 = 0. \tag{1}$$

The tangent to (1) at the point $(y_1:y_2:y_3)$ is given by

$$c_1x_1y_3 + c_2x_2y_2 + c_1x_3y_1 = 0. \tag{2}$$

Let $[u_1:u_2:u_3]$ be the line coordinates representing the line

$$u_1x_1 + u_2x_2 + u_3x_3 = 0,$$

then $[c_1y_3:c_2y_2:c_1y_1]$ are coordinates of the line (2).

It is easy to show that the equation of the conic (1) in line coordinates is

$$2c_2u_1u_3 + c_1u_2^2 = 0. \tag{3}$$

Let $[a_1:a_2:a_3]$ be the line AB, where we have

$$2c_2a_1a_3 + c_1a_2^2 = 0 \tag{4}$$

since AB is also tangent to the conic, and let P be the point $(z_1:0:z_3)$. Then A and B are the points $(a_2:-a_1:0)$ and $(0:a_3:-a_2)$. The coordinates of AP and BP are $[-a_1z_3:-a_2z_3:a_1z_1]$ and $[a_3z_3:-a_2z_1:-a_3z_1]$ respectively. Then the points X and Y are $(0:a_1z_1:a_2z_3)$ and $(a_2z_1:a_3z_3:0)$ whence the coordinates of XY are $[a_3z_3^2 : -a_2z_1z_3 : a_1z_1^2]$. Using equation (4) we see that the coordinates of XY satisfy equation (3), from which we see that the envelope of XY is the original conic.

A Note by the Editors. The theorem of this problem is the converse of that of 3458 [1930, 508], a proof of which is given [1931, 350] by analytical methods and

also by the use of Brianchon's theorem. The truth of one theorem follows easily from that of the other.

Another synthetic proof is as follows. As P moves along DE, the range of points X generated on BC is projective with the range Y on AC. Hence XY envelopes a conic. When P is at D, we have AC tangent to the envelope at D, and similarly BC is tangent to it at E. When P is at G, the intersection of DE with AB, we have AB tangent to the envelope. This suffices to show that the envelope is the original conic. The point of tangency F of AB with the envelope is easily seen to be the harmonic conjugate of G with respect to A, B.

Also solved by Rufus Crane, R. Goormaghtigh, J. D. Hill, A. Pelletier, A. W. Randall, Wallace Smith, H. E. Stelson, G. A. Yanosik, F. L. Wren, Kamcheung Woo, and the Proposer.

3474 [1931, 51]. *Proposed by R. E. Gaines, University of Richmond, Va.*

From a point P on the ellipse $b^2x^2+a^2y^2=a^2b^2$ two chords are drawn, one through $(ae, 0)$ and the other through $(-a, 0)$; for what value of e and what position of P will these two chords trisect the area of the ellipse?

Solution by A. Pelletier, Montreal, Canada

Let O be the center of the ellipse; $A'A$, its major axis; PA' and PC, the required chords, where PC passes through the focus F on OA. Consider the circle on $A'A$ as a diameter which projects orthogonally into the ellipse. Then the points P' and C' on the circle which project into P and C must be such that $P'A'$ and $P'C'$ trisect the area of the circle. The angle $P'OA=\alpha$ is the eccentric angle of the required point P. The area enclosed by the arc $P'A$ of the circle and its two chords AA', $A'P'$ is equal to $\frac{1}{2}\pi a^2-\frac{1}{3}\pi a^2=\frac{1}{6}\pi a^2$. It is also equal to $\frac{1}{2}a^2\alpha+\frac{1}{2}a^2\sin\alpha$, and hence

$$\alpha+\sin\alpha=\frac{\pi}{3}. \tag{1}$$

The angle α is approximately .53627 or 30°43′33″. The coordinates of P are $(a\cos\alpha, b\sin\alpha)$.

Obviously $A'P'=C'P'$, and hence $P'O$ bisects the angle $A'P'C'$. Then $\angle OA'P'=\angle OP'A'=\angle OP'F=\frac{1}{2}\alpha$; also $\angle AFP'=3\alpha/2$. Hence

$$e=\frac{OF}{OA'}=\frac{P'F}{P'A'}=\frac{\sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{3\alpha}{2}\right)}=\frac{1}{1+2\cos\alpha}=.36775 \text{ approximately}. \tag{2}$$

Also solved by R. Goormaghtigh, O. J. Ramler, F. L. Wilmer, and the Proposer.

MATHEMATICS CLUBS

EDITED BY F. M. WEIDA, The George Washington University, Washington, D. C.

Send all reports of club activities to F. M. Weida, The George Washington University, Washington, D. C. All manuscript should be typewritten, with double spacing, and with margins at least one inch wide.

CLUB ACTIVITIES

A.

The Pi Mu Epsilon Mathematical Fraternity

Pi Mu Epsilon is an academic fraternity in institutions of university grade. Its primary aim is the advancement of mathematics and scholarship. It is a living, active, working fraternity of scholars in which the members are actively engaged in study and research in the preparation of papers in the field of mathematical science to be presented at its regular meetings.

CHAPTER REPORTS

1930–1931

Alpha of Iowa, Iowa State College

The officers for 1930–1931 were: Frances L. Fish, Director; Richard Apple, Vice Director; Donald Paul Needham, Treasurer; Pauline Evarts, Secretary; E. C. McCracken, Instructor in Physics, Librarian; Professor D. L. Holl, Faculty Advisor; Dr. E. R. Smith, Head of the Department of Mathematics, Permanent Secretary.

We now have sixty five active members. We had two initiations the past year, one on November 25th, at which time we took in ten new members, and the other on May 12th, when we initiated ten.

At the annual college Honor's Day in the fall, 1930, we awarded a check of $25 and a certificate to Henry Dale Bossert, a junior architectural engineering student. We give this prize each year to the student having the highest scholastic average who has completed the course in calculus. He is also taken into Pi Mu Epsilon.

The meetings and programs were as follows:

October 28, 1930: "Analytic solutions of differential equations" by Dr. Holl.

November 25, 1930: Initiation and banquet.

December 10, 1930: "Discontinuous functions" by Burnette Backhaus; "An existence theorem" by Robert Cochran.

January 28, 1931: "The use of mathematics in other sciences" by Miss Colpitts; "Review of the life and works of Steinmetz" by George Gross.

February 19, 1931: "Projective Geometry" by Mr. Jean Hempstead.

March 11, 1931: "Expressions of an integral in terms of some differential operators" by Mr. L. F. Robertson.

April 22, 1931: "A study of the value of i^i and its historical significance" by Bernice Brown.

May 12, 1931: Initiation and banquet.

June 4, 1931: "Mathematical derivation of the formula describing the motion of Foucault's pendulum" by Charles Wells.

FRANCES L. FISH, *Director*

Pi Mu Epsilon of the University of Montana

The Montana chapter of Pi Mu Epsilon has been enjoying a successful year. Meetings have been held regularly once a month.

The following officers for the year 1930–1931 were elected on June 4, 1930: Paul Treichler,

Director; Albert Besancon, Vice Director; Elsie Magnuson, Secretary; Professor G. D. Shallenberger, Treasurer.

Initiation for the new members was held at the annual banquet on March 17, 1931. The following took the pledge of membership: Kathryn Coe, '33; Emma Bravo, '33; Horace Worden, '33; Joe Lasby, '33; Franklin Long, '32; Gene Sunderlin, '32; William White, '30; C. Smith, '28.

There are now thirty active members. As a partial requirement for membership, certain papers have been read before Pi Mu Epsilon.

The meetings and programs were as follows:

November 12, 1930: "De Moivre's theorem" by Franklin Long.
November 19, 1930: "Table of integrals" by William White.
December 3, 1930: "Higher plane curves" by Kathryn Coe.
December 10, 1930: "History of calculus" by Professor N. J. Lennes.
January 14, 1931: "Rotation of axes" by Horace Worden.
January 28, 1931: "Historical development of plane analytic geometry" by Emma Bravo.
February 4, 1931: "Sets of axioms" by William White.
February 11, 1931: "Cubic and biquadratic" by Dex Chevalier.
February 18, 1931: "Fictions in thought" by Burr Lennes (a report of *Philosophy as If* by Vathinger).
February 25, 1931: "Quadratic functions in two variables," "Discriminant," by M. Monaco.
March 11, 1931: "Review of *Pastures of Wonder* by Keyser" by Gene Sunderlin.
April 8, 1931: "Involutes and evolutes" by Dick Thomas.
May 20, 1931: "Probability" by C. Smith.

The annual banquet was held on March 17, the birthday of the Montana chapter. A picnic was held on April 26.

ELSIE MAGNUSON, *Secretary*

Pi Mu Epsilon of the University of Kentucky

The officers of the Kentucky Alpha chapter of Pi Mu Epsilon for the year 1930–1931 were as follows: Professor H. H. Downing, Director; Assistant Professor M. C. Brown, Vice Director; Assistant Professor D. E. South, Treasurer; Associate Professor F. E. LeStourgeon, Secretary; Instructor Sallie Pence, Librarian.

The chapter has twenty-seven active members. Four new members were taken in during the year. Initiation of new members was held on December 18, and again on May 7. During the year there were, besides the eight regular meetings at which papers were read, three business meetings, and a banquet on the evening of May 7 in honor of the initiates.

The meetings and programs were as follows:

October 23, 1930: "A formula in analytic geometry" by Professor C. G. Latimer.
November 20, 1930: "Curvature of Einstein space" by Professor E. L. Rees.
February 19, 1931: "Discontinuous functions" by Professor F. E. LeStourgeon.
February 26, 1931: "Cosines of multiple angles in terms of cosines of the simple angles" by Professor H. H. Downing.
March 26, 1931: "Contingent functions" by Professor D. E. South.
April 15, 1931. "The course of deep bore holes" by Professor O. T. Koppius of the department of physics.
April 30, 1931: "Newton's quadratures of curves" by Mr. R. C. Bullock.
May 7, 1931: "Algebraic geometry (Something of its methods and results)" by Professor Mayme I. Logsdon of the University of Chicago.

ELIZABETH LESTOURGEON, *Secretary*

Alpha of Oklahoma, University of Oklahoma

This Alpha Chapter of Pi Mu Epsilon reports the following officers for the year 1930–1931 who have held office since May, 1930: Mr. Earl LaFon, Director; Mr. John C. Brixey, Vice Di-

rector; Miss Dora McFarland, Secretary; Mr. Charles C. Edmondson, Treasurer; Professor J. O. Hassler, Librarian.

Of the thirty five members on the roll of this chapter, fourteen are faculty members, ten are graduate students in mathematics or natural science, nine are seniors, and two are juniors. Two elections were held during the year, at which sixteen new members were admitted; six in November and ten in April. Each time an initiation dinner was held at the Faculty Club. These dinners took the place of the regular meetings and were the only social meetings of the year.

Regular meetings were held on the second and fourth Thursdays of each month. During the year the following papers and discussions were given:

October 23, 1930: "The three-cusped hypocycloid" by Mr. Springer.

November 13, 1930: "Topics in the theory of numbers" by Mr. Ransbarger, Mr. Edmondson, Mr. LaFon.

December 1, 1930: "A historical discussion of the quadratic, cubic, and quartic equations" by Mr. Dorsett.

January 8, 1931: "Certain canonical forms and their properties for quadratic, cubic, and quartic equations" by Dr. Elsie McFarland.

February 26, 1931: "Topics from continued fractions" by Mr. John Brixey.

March 1, 1931: "Tangential equations" by Mr. Springer.

March 26, 1931; "Fallacies in recent attempts to trisect an angle" by Mr. Guthrie.

April 9, 1931: "Some differential geometry by vector analysis" by Mr. LaFon.

April 23, 1931: "On the nature of projective differential geometry" by Dr. Hassler.

DORA MCFARLAND, *Secretary*

Beta of Missouri, Washington University

The Missouri Beta chapter reports a very successful year under the following officers elected at the meeting held on May 17, 1930:

Professor Edmond Siroky, Director; Elizabeth Harris, Southwestern Bell Telephone Co., Vice Director; Professor Jessica M. Young, Secretary; Harold K. Crowder, Assistant Secretary; Instructor Bayard R. Brick, Treasurer; Ford Pennell, Librarian.

We also have an executive committee. The student members of this committee were: Philip Mills Arnold, Lillian Hoagland, Roland E. Miller, Helen Stammer.

There were fifty three active members during the year 1930–1931. Thirty new members were initiated on April 25, 1931, distributed as follows: from the College of Liberal Arts—eight; Schools of Engineering and Architecture—twenty; School of Graduate Studies—two.

The ten meetings and programs held during the year were as follows:

October 9, 1930: "Old text books in our library" by Philip M. Arnold; "Mathematical puzzles" by John C. Lebens.

November 4, 1930: "The algebra of logic" by Charles O. Quade; "Vector algebra" by Richard Torrence.

December 8, 1930: "Diffraction patterns" by Charlotte Wieghard; "The fourth dimension" by Lillian Hoagland.

January 8, 1931: "The Arabians and mathematics" by Rosella Dodt; "Mechanical integration" by William E. Stephens. (This paper was illustrated with models.) Mr. Stephens's paper was the prize paper for the year.

February 10, 1931: "Generating functions" by Professor Otto Dunkel; "The empty column" by Walter W. Schmidt.

March 9, 1931: "Some mathematical recreations" by F. Richard Singer; "Balmer series" by Jessie Best.

March 18, 1931: Passing of amendment to by-laws raising the initiation fee to $5.00. Election of new members.

April 13, 1931: (Special meeting.) An open meeting held in conjunction with the Washington University Chapter of the Society of the Sigma Xi. Professor J. A. Schouten, of the Delft

Technical School in Holland, gave a popular lecture on "Generalized idea of parallelism and applications to modern geometry and physics." About 200 persons attended the lecture and the reception following. Four guests, including Director General Ingold, honored us by coming down from the Missouri Alpha Chapter.

April 25, 1931: Initiation and annual banquet; "Some theorems related to the incenters and excenters of triangles" by Instructor Pearl Colby Miller.

May 16, 1931: Business meeting and social gathering. Award of a prize of $10.00 to William E. Stephens for the paper, presented before the chapter during the year, that was of most general interest to student members. Treasurer's report. Election of officers and student members of executive committee for the year 1931–1932. Refreshments and a social hour were enjoyed following each meeting. The students showed a marked interest in the meetings throughout the year. The program committee (Professor Eugene Stephens, Chairman) should be congratulated on having so many papers presented by students.

J. M. YOUNG, *Secretary*

Alpha of Oregon, The University of Oregon

The officers for 1930–1931 were: Mildred M. Wharton, Director; Arthur Johnson, Vice Director; Helen Elliott, Secretary; Robert Holmquist, Treasurer; Kenneth Kienzle, Executive Chairman.

The officers were elected May 28, 1930 by secret ballot. The organization had thirty seven active members in 1930–1931. Three members of the mathematics faculty and nine graduate assistants were included in that number.

The primary aim of Pi Mu Epsilon has become the aim of our organization, namely: "scholarship for the individuals in the organization in all subjects and particularly in mathematics."

A student to be elected to membership must be in the upper half of his class in general scholarship, and in the upper third in mathematical scholarship. He must have completed at least two terms of differential and integral calculus and be taking his final term of the subject. Members of the mathematical faculty, major and minor graduate students in mathematics and major and minor undergraduate students in mathematics subject to the above requirements are eligible.

The meetings and programs were as follows.

November 24, 1930 "Mathematics and music" by Kenneth Kienzle.

January 12, 1931: "Discussion and presentation of the mathematical principles involved in the use and working of a calculating machine" by Edna Keepers.

February 9, 1931: "Presentation and discussion of the basis, history and properties of non-Euclidean Geometry" by Dr. R. R. Davis.

April 6, 1931: "Mathematics and its relation to biological growth" by George Schlesser.

On October 30, 1930 and on May 14, 1931, the club held business meetings. On May 19, 1931, the Oregon Alpha chapter of Pi Mu Epsilon was installed.

Throughout the year our attention has been centered about our petition to the national honorary fraternity. In April we received word from Professor Louis Ingold of the University of Missouri of the granting of our charter; and on May 19, twenty-nine charter members were installed in the Oregon Alpha chapter. At that time, twenty-one new members, graduate assistants and faculty members were initiated also. The President of our University, Dr. Arnold Bennett Hall, addressed us at a banquet following our ceremony.

The year will be closed with a picnic for all new and old members.

MILDRED M. WHARTON, *Director*

B.

LOCAL MATHEMATICS CLUBS

The Mathematics Club of Boston University

The mathematics club of Boston University aims to promote good fellowship among the students interested in mathematics, to encourage the study of mathematics in the University, and to

discuss the more interesting aspects of mathematics. Any person interested in higher mathematics is eligible for membership. The club has about thirty active members.

The officers are students in the College of Liberal Arts who are majoring or minoring in mathematics. Elections are held at the April meeting by ballot and by majority vote. The officers for the year 1930–1931 were: Alden P. Cleaves, President; Muriel M. Sutherland, Vice President; Marion L. Goodwin, Secretary; Samuel Feldman, Assistant Secretary; Warren C. Dean, Treasurer; Ruth H. Schelin, Assistant Treasurer; Carleton H. Foss, Gladys E. Knowlton, Lucien B. Taylor, Executive Committee.

Regular meetings are held the first and third Thursday noons of each month. The speakers at these meetings are usually students.

The meetings and programs were as follows:
October 2, 1930: "Mathematics in pure economics" by Professor Robert E. Bruce.
October 16, 1930: "Mathematics in nature" by Ruth Schelin.
November 6, 1930: "Properties of parabolas geometrically" by Gladys Knowlton.
November 20, 1930: "Mathematics of Thales" by Warren Dean.
December 4, 1930: "Life of Euler" by Doris Atkinson.
January 8, 1931: "Projective ornaments" by Ora Park.
February 19, 1931: "Geometrical fallacies" by Thomas Homkowycz.
March 5, 1931: "Life of Descartes" by Francis Blackwell.
March 19, 1931: "Paper folding" by Carleton Foss.
April 16, 1931: "Living mathematics" by Eleanor Johnson.
April 30, 1931: "The mathematics of field artillery" by Professor Warren O. Ault.

The annual five dollar prize for the best paper was awarded to Thomas Homkowycz. Honorable mention was given to Carleton Foss and Gladys Knowlton.

Each semester the club holds a supper, social, and theatre party. The first was held on Wednesday, December 10. The entertainment, following the supper, consisted of music, card tricks, and a talk by Professor Bruce about his trip to Angcor, the ancient capital of Cambodia (illustrated with slides). The play, "Street Scene" concluded the evenings program. At the second party, held on Tuesday, April 7, Professor Bruce spoke on "Early numerals in India" and exhibited several snapshots which he had taken. Miss Mildred B. Mitten, '24, one of the charter members of the club, was present and told us a little of the early activities of the club.

The club affords an excellent opportunity for informal acquaintance between students and teachers.

MARION L. GOODWIN, *Secretary*

The Mathematics Club of the New Jersey College for Women

The officers for 1930–1931 were: Beatrice Velten, President; Louise Killheffer, Vice President; Helen Carpenter, Secretary; Marian Bruen, Treasurer. These officers were nominated by a nominating committee consisting of the officers of the preceding year and voted upon by the members of the club at a regular meeting in May 1930.

The aim of the club is to stimulate interest in the field of mathematics and to afford opportunities for investigation. Membership is limited to those who have taken or are taking a course in calculus. Our present membership is thirty-five.

The formal papers for the year were: "Modern discoveries in geometry with respect to the orthopole" by Dr. Richard Morris, March, 1931; "Graphical methods in analysis" by Mary Crandall, May, 1931; "Power series" by Beatrice Velten, May, 1931.

The topics discussed by student members were: "Babylonian mathematics" by Lucy Macaluso, October, 1930; "Egyptian mathematics" by Louise Killheffer, October, 1930; "Greek mathematics" by Marie Kozesnick, October, 1930; "Preservation of mathematics by Hindus and Arabians" by Doris Swain, November, 1930; "Apollonius, Euclid, and Archimedes" by Helen Shack, November, 1930; "Beginnings of analytic geometry" by Verna Wilson, November, 1930; "Leibnitz's and Newton's controversy on calculus, by Caroline Jung, November, 1930; "Modern

geometry" by Lillian Solomon, November, 1930; "Archimedes' proof of the area of a segment of a parabola" by Beatrice Velten, January, 1931; "Determination of an angle of a right triangle without the use of tables" by Marian Bruen, February, 1931; "How to inscribe a rectangle within another rectangle" by Margaret Thompson, February, 1931; "Proof of Morley's theorem relating to a triangle" by Janet Mather and Virginia Stevens, March, 1931; "Proofs and verification of some area formulas of the triangle" by Edna Schnitzler, Mae Blydenburgh, Irene Stevens, Alice Maier, Helen Shack, Margaret McCallum, March and April, 1931; "Groups" by Marian Bruen, April, 1931; "Applications of groups to trigonometry" by Irene Stevens, April, 1931.

The other activities of the club were: A mathematics club tea in October, 1930; a Christmas party in December, 1930; a mathematics club social and dance in May, 1931; and joint dinners of the two mathematics clubs in December, 1930 and April, 1931.

We also had lectures by visiting speakers. The speakers and the topics discussed were: Professor R. C. Archibald of Brown University spoke on "Mathematics prior to the Greeks" in December, 1930; Dr. S. A. Schelkunoff of the Bell Telephone Laboratories spoke on "Telephony and the application of mathematics to some of its problems" in April, 1931.

HELEN H. CARPENTER, *Secretary*

The Mathematics Club of the University of Buffalo

This club was organized in September, 1929 for the purpose of stimulating an interest in the history and problems of mathematics. All students majoring in mathematics and others interested are eligible for membership in the club. We had twenty-five members. Miss Margaret Morgan was our president.

The meetings and programs were as follows:

October 1930: Organization and election of officers; "Review of 'Flatland' by A. Square" by Miss Montague.

November 1930: "Squaring the circle" by Miss Agnes Higgins; "Magic squares" by Dr. Gehman.

February 1931: "Short cuts in computation" by Miss Montague and Professor Harrington.

April 1931: "The fourth dimension" by Mr. John Greenwood of Technical High School.

HARRIET F. MONTAGUE

The Mathematics Club of Wellesley College

The aim of the mathematics club for the year 1930–1931 has been no different from that of other years—that is, it has been to bring Wellesley's students of mathematics into more intimate contact socially pleasurable and instructively valuable. Membership in the club is open to all students of mathematics with the exception of Freshmen. This year the active members number sixty-eight.

The officers for this year, elected by ballot last May 20, 1930 at the last meeting of the club for that year, were:

Melita Holly, President; Katherine Atwood, Vice President; Virginia Francis, Treasurer; Emily Neal, Secretary; Claudia Jessup, Junior Executive; Miss Lennie P. Copeland, Faculty Advisor.

Unfortunately, the president was forced to resign after Christmas because of ill health. Since then, Katherine Atwood has been acting president.

The year's programs have been interesting and varied. In the course of the meetings, a number of topics have been discussed by student members of the club.

The meetings and programs were as follows:

October 17, 1930: "Mathematics and botany" (illustrated with drawings) by Barbara Little; "Probability" by Frances Fletcher; "Numbers, an explanation of a system based on twelve instead of ten" by Barbara Trask.

November 14, 1930: "Number systems" by Esther Van Artsdalen; "Codes and ciphers" by Barbara Bicknell; "Zeno's paradoxes" by Claudia Jessup; "The role of '*e*' in Physics" by Miss Gabriel Asset, Assistant in the Department of Physics.

December 5, 1930: The program was presented by Miss Helen Merrill and Miss Marion Stark, professors of mathematics here, who entertained the club with false mathematical proofs, etc., and ended their program with Miss Stark's reading some mathematical poetry; an article, "Some excitement in mathematics" by Stephen Leacock; and the story of A, B, and C—"The human element in mathematics."

The club ended its year's work on April 17, 1931 with the presentation of a play—"Modern mathematics looks up his ancestors"—written in verse by Miss Marion Stark. To the play were invited not only members of the club but all in the College interested in mathematics.

EMILY NEAL, *Secretary*

NOTES AND NEWS

Readers are invited to contribute to the general interest of this department by sending items to Professor H. W. Kuhn, Ohio State University, Columbus, Ohio.

The Rhind Mathematical Papyrus is a magnificent work and should be in every college library. The edition is absolutely limited and no more will be available when this edition is exhausted. The price of $15.00 to members of the Association and $20.00 to non-members has been found inadequate to cover the expense of storage, commissions, postage, etc. Therefore, after January 1, 1932, the price to members will be $20.00 and to non-members $25.00, postage prepaid. Orders received before that date will be filled at the old prices. Members, individual and institutional, should order directly through Secretary W. D. Cairns, Oberlin College, Oberlin, Ohio. Non-members should order through the Open Court Publishing Co., 339 East Chicago Avenue, Chicago, Illinois.

Professor W. B. Carver, of Cornell University, will be the editor-in-chief of this Monthly beginning with the January, 1932 issue. From now on all manuscripts submitted for publication should be sent to him at White Hall, Cornell University, Ithaca, N. Y.

Assistant Professor C. T. Bumer, of the Ohio State University, has been appointed assistant professor of mathematics at Kenyon College.

Assistant Professor Evelyn T. Carroll, of Wells College, has been promoted to an associate professorship of mathematics.

Associate Professor H. H. Downing, of the University of Kentucky, has been promoted to a professorship of mathematics.

Dr. W. W. Flexner has been appointed lecturer in mathematics at Bryn Mawr College.

Dr. H. C. Gossard, a charter member of the Mathematical Association, was inaugurated as president of the New Mexico Normal University on October 8, 1931.

Dr. Deborah M. Hickey has been appointed professor of mathematics at the Mississippi Delta State Teachers College.

Mr. E. H. Hildebrandt has been appointed assistant professor of mathematics at De Pauw University.

Professor Temple R. Hollcroft, of Wells College, will spend his sabbatical year, 1931–32, in travel and study abroad.

Assistant Professor W. G. Hubert has been promoted to an associate professorship at the College of the City of New York.

Mr. Lonnie Langston has been promoted to an assistant professorship of mathematics at the Texas Technological College.

Associate Professor H. F. MacNeish, of the College of the City of New York, has been transferred to Brooklyn College.

Dr. Gertrude I. McCain, of East Radford, Virginia, has been appointed professor of mathematics at Marymount College, Salina, Kansas.

Dr. J. E. Merrill has been appointed assistant professor of astronomy at the University of Illinois.

Dr. A. K. Mitchell, of Yale University, has been appointed to an assistant professorship at Trinity College, Hartford.

Professor U. G. Mitchell, of the University of Kansas, has been appointed head of the department of mathematics, succeeding Professor C. H. Ashton who asked to be relieved of administrative duties because of ill health.

Assistant Professor L. T. Moore, of Yale University, has been appointed assistant professor at Brooklyn College of the City of New York.

E. J. Oglesby, of Washington Square College, New York, has been appointed professor of engineering mathematics in the University of Virginia.

Dr. Anna Pell-Wheeler, will resume her position as head of the department of mathematics at Bryn Mawr College in the fall of 1931.

Assistant Professor G. A. Pfeiffer, of Columbia University, has been promoted to an associate professorship of mathematics.

Z. M. Pirenian, instructor at Alabama Polytechnic Institute, has been appointed assistant professor at the University of Florida.

Professor C. L. Poor, of the department of celestial mechanics of Columbia University, has retired.

Associate Professor J. F. Ritt, of Columbia University, has been promoted to a professorship of mathematics.

Dr. Hazel E. Schoonmaker has been appointed professor of mathematics at Hartwick College, Oneonta, N. Y.

Dr. G. W. Starcher, of Ohio University, has been promoted to an assistant professorship of mathematics.

Assistant Professor J. S. Taylor, of the University of Pittsburgh, has been promoted to a professorship of mathematics.

Assistant Professor V. B. Teach, of the Armour Institute of Technology, has been promoted to an associate professorship of mathematics.

Dr. Earl Thompson has been promoted to an associate professorship of the teaching of mathematics at the Texas Technological College.

Associate Professor Bird M. Turner, of West Virginia University, has been promoted to a professorship of mathematics.

Dr. R. S. Underwood has been promoted to a professorship of mathematics at the Texas Technological College.

Dr. A. Marie Whelan has been promoted to an assistant professorship of mathematics at Hunter College.

Dr. S. D. Zeldin has been promoted to an assistant professorship of mathematics at the Massachusetts Institute of Technology.

The following appointments to instructorships in mathematics are announced:

Brooklyn College, Miss Margaret M. Young.
Brown University, Mr. Max Astrachan.
University of Chicago, Dr. C. W. Mendel, Dr. W. T. Reid.
Columbia University, Dr. A. C. Berry.
Cornell University, Mr. A. H. Black.
Hunter College, Miss Polly P. Nelson.
University of Kansas, Miss Eula Johnson, Miss Winona Venard.
Long Island University, Mr. K. G. Fuller.
Marquette University, Mr. Stephen Lewandowski.
Ohio State University, Dr. Laurens Earle Bush.
Princeton University, Mr. J. L. Vanderslice.
Stanford University, Mr. H. M. Bacon.
Trinity College, Mr. W. H. Mitchell.
Wells College, Miss Nancy Cole.

Professor A. A. Michelson, of the University of Chicago, the distinguished physicist, died May 9, 1931, at the age of seventy-eight.

Dr. Bessie I. Miller, formerly professor of mathematics at Rockford College, and a charter member of the Association, died February 4, 1931.

Professor Claude Irwin Palmer, head of the Department of Mathematics and Dean of the Students at the Armour Institute of Technology, died April 9, 1931, at the age of sixty. Professor Palmer was a charter member of the Mathematical Association of America.

Advance in Price of the Rhind Mathematical Papyrus

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Individual and institutional members may procure copies at $15.00 per set through Secretary Cairns at Oberlin, Ohio. All others must order through the Open Court Publishing Company, 339 E. Chicago Avenue, Chicago, Ill., at $20.00 per set. These prices will be advanced to $20.00 and $25.00, respectively, after January 1, 1932.

IT IS A MAGNIFICENT WORK and should be in every college library. The edition is absolutely limited. One-third of the sets are already sold, and no more will be available when this edition is exhausted. Order now and save the added cost in 1932.

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Legal Form for Gifts and Bequests

I hereby give[1] to the Board of Trustees of the Mathematical Association of America the sum of.....Dollars, to be known as the.....Fund, and to be used for[2] { Endowment—the income only of which may be expended. / Special Projects—for which both principal and income may be expended. }

Witness:..... Signature.....

[1] In case of a bequest, the first line should read "I hereby give and bequeath," etc.
[2] Indicate which one of the two purposes is desired, and omit the other.

The Association needs funds for scientific publications and for the promotion of scientific activities.

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DIRECTORY

EDITORIAL CORRESPONDENCE should be addressed to the Editor-in-Chief, W. H. Bussey, 106 Folwell Hall, University of Minnesota, Minneapolis, Minn.

BOOKS FOR REVIEW should be addressed to R. A. Johnson, Brooklyn College, 66 Court Street, Brooklyn, N.Y.

BUSINESS CORRESPONDENCE should be addressed to the Secretary-Treasurer of the Association, W. D. Cairns, 33 Peters Hall, Oberlin, Ohio.

CHANGE OF ADDRESS: Members should send notice of any change of address to the Secretary-Treasurer, W. D. Cairns, Oberlin, Ohio, before the 10th of each month.

MEETINGS OF THE ASSOCIATION AND ITS SECTIONS

Fifteenth Summer Meeting of the Association, Minneapolis, Minnesota, Sept. 7-8, 1931.

The following is a list of the sections of the Association, with dates of those section meetings which have been scheduled for 1931.

Illinois, Peoria, May 1-2.
Indiana, Muncie, May 1-2.
Iowa, Davenport, May 1-2.
Kansas, Topeka, Jan. 24.
Kentucky, Lexington, May 9.
Louisiana-Mississippi, Natchitoches, La., March 13-14.
Maryland-District of Columbia-Virginia, Richmond, Va., May 9.
Michigan, Ann Arbor, March 21.
Minnesota, St. John's University, Collegeville, May 16.
Missouri, St. Louis, November.
Nebraska, Lincoln, May 8.
Ohio, Columbus, April 2.
Philadelphia, Philadelphia, Nov. 28.
Rocky Mountain, Boulder, Colo., April 17-18.
Southeastern, Auburn, Ala., April 24-25.
Southern California, Occidental College, Los Angeles, March 21.
Texas, Fort Worth, Jan. 31.

Affiliated Organizations: The New England Association of Teachers of Mathematics, The National Council of Teachers of Mathematics.

PREFATORY NOTE

This Catalog is published for the information of the members of the Association. Many are so situated that they do not have easy access to certain books and periodicals which are available in the Association Library; and they are invited to make free use of the available material, under the liberal rules which are printed below. Small though the Library is as yet, it will be found to contain many items that have significance for persons interested in collegiate mathematics.

The arrangement of this Catalog follows that of the Catalog of the American Mathematical Society, there being two sections, the first containing titles of Periodicals and Publications of Governments, Learned Societies, Educational Institutions, etc.; and the second section containing the titles of other books and pamphlets. The first group is arranged according to the *Union List of Serials*, Gregory, 1927 (sponsored by the American Library Association).

LIBRARY RULES

1. Applications for books may be made in person or by letter to W. D. CAIRNS, Secretary-Treasurer, 33 Peters Hall, Oberlin, Ohio. To these applicants books will be sent by express at the expense of the borrower.

2. A book may be kept four weeks from the date of leaving the library, but the loan may generally be renewed by writing to the Secretary-Treasurer in advance of the due date.

3. Borrowers should return books in person or by prepaid express.

I. PERIODICALS, GOVERNMENT PUBLICATIONS, AND PUBLICATIONS OF SOCIETIES, ACADEMIES, UNIVERSITIES, TECHNICAL SCHOOLS

R. Accademia Nazionale dei Lincei, Rome.
Atti, s6, v1, n9+, 1924+ (v7 n1, v9 n12 missing)
Acta Litterarum ac Scientiarum. *See* Szeged. Tudomány-Egyetem, Acta.
Acta Mathematica, Stockholm. 17+, 1893+
American Journal of Mathematics, Baltimore. 38 n3+, 1916+
(v39 n3 missing)
American Mathematical Monthly, Oberlin. 1+, 1894+
American Mathematical Society, New York.
Bulletin, v6+, 1899+ Indexes; 1891–1904, 1904–1914, 1914–1924.
Catalogue of the Library, 1910, 1926.
Colloquium lectures. See Colloquium publications.
Colloquium publications, 1+, 1903+ (1. Boston (1903) fourth colloquium, H. S. White, Linear systems of curves on algebraic surfaces; F. S. Woods, Forms of non-euclidean space; E. B. Van Vleck, Selected topics in the theory of divergent series and continued fractions, 1905. 2. New Haven (1906) fifth colloquium, E. H. Moore, Introduction to a form of general analysis; E. J. Wilczynski, Projective differential geometry; M. Mason, Selected topics in the theory of boundary value problems of differential equations, 1910. 3. Princeton (1909) sixth colloquium, G. A. Bliss, Fundamental existence theorems; E. Kasner, Differential-geometric aspects of dynamics, 1913. 4. Madison (1913) seventh colloquium, L. E. Dickson, On invariants and the theory of numbers; W. F. Osgood, Topics in the theory of functions of several complex variables, 1914. 5. Cambridge (1916) eighth colloquium, G. C. Evans, Functionals and their applications, selected topics including integral equations; O. Veblen, Analysis situs, 1918 and 1922. 6. G. C. Evans, The Logarithmic potential. Discontinuous Dirichlet and Neumann problems, 1927. 7. E. T. Bell, Algebraic arithmetic, 1927. 8. L. P. Eisenhart, Non-Riemannian geometry, 1927. 9. G. D. Birkhoff, Dynamical systems, 1927. 10. A. B. Coble, Algebraic geometry and theta functions, 1929. 11. D. Jackson, The theory of approximation, 1930. 12. S. Lefschetz, Topology, 1930.)
F. Klein, Lectures on mathematics delivered at Evanston in 1893. Republished in 1911.
Mathematical papers read at the International Mathematical Congress at Chicago in 1893. New York, 1896.
Registers, 1916–1918, 1926, 1928.
Transactions, 1+, 1900+
American Philosophical Society, Philadelphia.
Proceedings, 54+, 1915+ (Various numbers missing)
American Physical Society. Physical Review. *See* Physical Review.
Physics, *See* Physics.
Annali delle Universita Toscane, Pisa. n s v4–10, 1919–1926.
Annali di Matematica pura et applicata, Bologna. s4, v1+, 1923+
General index.
Annals of Mathematics, Princeton, N. J. s2 v12+, 1910+
Annuario di Catania, *See* Circolo Matematico di Catania. Annuario.
Archives de Mathématiques pures et appliqués, Liège. v1 n1, Jl. 1907‖
Association des Professeurs de Mathématiques, Paris.
Bulletin, 1+, 1910+
Suppléments, 1924+

Benares Mathematical Society, Benares, India.
Proceedings, v1+, 1919+ (v10 missing)
Bollettino di Bibliografia e Storia delle Scienze Matematiche, Turin.
18–19, 1916–1917; s2 v1–2, 1918–1921.
Bollettino de Matematica (Conti). With Sezione Storico-Bibliografia (continuation of Loria's Bollettino di Bibliografia e Storia della Scienze Mathematiche) Bologna. n s v1+,1922+
Brown University, Providence, R. I.
Bulletin of the Department of Mathematics, n1. (Suggestions for students of mathematics; mathematics and life activities, 1922.)
Brünn. Masaryk Université. Faculté des Sciences.
Publications, 1921+ (Various numbers missing)
Buenos Aires. Universidad Nacional. Seminario Matemático Argentino.
Boletin, 1+, 1928+

Calcutta Mathematical Society.
Bulletin, 9+, 1917+ (v20 n2–4 missing)
California. University.
Publications in Mathematics, v1 n1. (H. W. Stager, On numbers which contain no factors of the form $p(kp+1)$, 1921.)

CARUS MATHEMATICAL MONOGRAPHS, 1+, 1925+ (n1, G. A. BLISS, Calculus of variations, 1925; n2, D. R. CURTISS, Analytic functions of a complex variable, 1926; n3. H. L. RIETZ, Mathematical statistics, 1927; n4. J. W. YOUNG, Projective geometry, 1930.)
CARUS MONOGRAPHS. *See* CARUS MATHEMATICAL MONOGRAPHS.
ČASOPIS PRO PĚSTOVÁNÍ MATHEMATIKY A FYSIKY, Prague. 57+, 1927+
CHRISTIAAN HUYGENS, Amsterdam. 1+, 1921+
CIRCOLO MATEMATICO, Palermo.
Annuario, 1914.
Rendiconti, 35+, 1913+
Supplemento, 11–12, 1919–1921; 15, 1926–1928.
CIRCOLO MATEMATICO DI CATANIA, Catania.
Annuario, 1921–1923.
See also ESERCITAZIONI MATEMATICHE; NOTE E MEMORIE DI MATEMATICA.
COLORADO COLLEGE.
Publications, 25, 51–52, 104. (25. F. H. LOUD, Notes on the computation of logarithms, 1907. 51–52. FLORIAN CAJORI, A history of the arithmetical methods of approximation to the roots of numerical equations of one unknown quantity, 1910. 104. C. H. SISAM, On non-ruled octic surfaces whose plane sections are elliptic, 1919.)

DEUTSCHE MATHEMATIKER-VEREINIGUNG, Leipzig.
Jahresbericht, 30+, 1921+ (v38 n1–4, v39 n1–4 missing)

EDINBURGH UNIVERSITY. Mathematical Department.
Research Papers, n2–3, 7. (n2. PIERRE HUMBERT, On some results concerning integral equations. n3. E. T. WHITTAKER, On the continued fractions which represent functions of Hermite and other functions defined by differential equations. n7. E. L. INCE. On certain theorems in continued fractions equivalent to Riemann's and other transformations of the *P*-function.)
ENGINEERING EDUCATION. *See* JOURNAL OF ENGINEERING EDUCATION.
ENSEIGNEMENT MATHÉMATIQUE, Paris. 18+, 1916+ (v18 n1, v20 n5–6 missing)
ESERCITAZIONI MATEMATICHE (Circolo Matematico) Catania. 1–3, 1921–1923.

FUNDAMENTA MATEMATICAE, Warsaw. 1+, 1920+

GIESSEN. UNIVERSITÄT. Mathematisches Seminar.
Mitteilungen, 1–15, 1921–1927.
GIORNALE DI MATEMATICA FINANZIARIA, Turin. 1+, 1919+ (v11 n1–5 missing)
GIORNALE DI MATEMATICHE (Battaglini) Naples. 53–56, 1915–1918.

HAMBURG. UNIVERSITÄT. Mathematisches Seminar.
Abhandlungen, 1+, 1921+

IBERICA, Barcelona. 3+, 1916+ (Various numbers missing)
ILLINOIS. UNIVERSITY.
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GROUP PHOTOGRAPH TAKEN AT THE MINNEAPOLIS MEETING, 1931

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THE FIFTEENTH SUMMER MEETING OF THE MATHEMATICAL ASSOCIATION

The fifteenth summer meeting of the Mathematical Association of America was held, by invitation, at the University of Minnesota, Minneapolis, Minn., on Friday, Monday and Tuesday, September 4, 7 and 8, 1931, in conjunction with the summer meeting and colloquium of the American Mathematical Society. Two hundred fifty-nine were present at the meetings, including the following one hundred forty-eight members of the Association.

H. M. ACKLEY, State Teachers College, Kalamazoo, Mich.
C. R. ADAMS, Brown University
L. K. ADKINS, State Teachers College, La Crosse, Wis.
NOLA LEE ANDERSON, Sophie Newcomb College
R. C. ARCHIBALD, Brown University
W. L. AYRES, University of Michigan

GLADYS L. BANES, Butler University
R. W. BARNARD, University of Chicago
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